On reliability estimation approaches for a Weibull failure modelling

Leoni L.*, Cantini A.*, De Carlo F.*, Tucci M.*

** Department of Industrial Engineering (DIEF), University of Florence, Viale Morgagni, 50135 – Florence – Italy (leonardo.leoni@unifi.it, alessandra.cantini@unifi.it, filippo.decarlo@unifi.it, mario.tucci@unifi.it)*

Abstract: During the last years, reliability analysis has attracted significant attention due to its vital role in risk and integrity management of hazardous installations. Indeed, estimating accurate probabilities of failure represents a crucial task for developing a cost-effective maintenance plan able to guarantee the safety of the operations. As a result, a sound tool capable of providing precise failure parameters is required. Traditional estimation approaches exploited for practical applications are the Maximum Likelihood Estimation (MLE) and the Least Square Estimation (LSE), while, quite recently, the advances in dedicated opensource software have led to a widespread use of Hierarchical Bayesian Modelling (HBM) for statistical purposes. The aim of this paper is to present and compare the application of the three aforementioned estimation methodologies in order to point out the most accurate one. A case study of five samples is considered to demonstrate and discuss the applicability of the frameworks, while a Weibull distribution is adopted to model the failure behavior of the studied devices. From this research, it emerged that the Bayesian inference is slightly more accurate than the other approaches. The outcomes of this study can help maintenance engineers and asset managers to adopt the most appropriate statistical tool for their analysis.

Keywords: Hierarchical Bayesian Modelling, Reliability analysis, Weibull distribution

1.Introduction

Reliability analysis has been receiving a great deal of attention thanks to its fundamental role in mitigating the risk arising from hazardous installations, such as process systems. Indeed, process plants usually deal with hazardous substances, whose loss of containment could produce severe consequences (Khakzad et al., 2013). Since failures are often regarded as the main source of dangerous scenarios (Jaderi et al., 2019), solid strategies to avoid the occurrence of failures need to be adopted. To this end, implementing maintenance policies is among the most renowned countermeasures to prevent failures.

Within the process of maintenance planning, developing a proper failure model for a given component is regarded as a crucial task. Indeed, estimating precise parameters characterizing the failure behaviour of equipment allows to avoid premature maintenance, without undermining safety issues. Nevertheless, providing accurate probabilities of failure is regarded as a difficult task due to limited data and vague features of failures (Yuhua & Datao, 2005). Consequently, many researchers have been working to reduce the uncertainties arising from the calculation (Abaei et al., 2018a; Khalaj et al., 2020; Purba, 2014; Purba et al., 2014; Witek, 2016). Quite recently Zhou et al. (2016) presented a methodology to estimate the failure likelihood of gas pipelines due to corrosion effects. The authors adopted a fuzzy approach to model two corrosion failure modes: corrosion thinning and corrosion cracking.

Reliability analysis, reliability assessment and probabilities of failure estimation have been executed utilizing different tools among which the most popular are Fault-Tree Analysis (FTA) (Javadi et al., 2011; Taheriyoun & Moradinejad, 2015), MLE (Huang & Dietrich, 2005; Odell et al., 1992) and Fuzzy logic method (Cheliyan & Bhattacharyya, 2018; MIRI et al., 2011). Despite their vast exploitation, the aforementioned techniques are non-updatable, moreover FT and Fuzzy logic are unable to consider multi-state variable and conditional dependencies. To solve these issues Bayesian Network (BN) has been adopted by many researchers for risk, safety and reliability analysis (Boudali & Dugan, 2005; Jia et al., 2021; Leoni et al., 2019; Li et al., 2019). Very recently, Sun et al. (2021) proposed a BN-based reliability framework for complex electronic system. In this work, a copula BN is used to model multivariate joint probability distributions, while physic failure simulations are exploited to develop for each node the fault distribution. A previous study developed by Taleb-Berrouane et al. (2020) proposed a dynamic approach for safety and reliability assessment, based on the integration between BN and Stochastic Petri Net. The developed methodology is capable to grasp the variation of safety and risk parameters, providing more flexibility.

Meanwhile, the advances in opensource software (e.g., OpenBugs), have led to a wider use of HBM (Spiegelhalter et al., 2007) for dealing with complex engineering problems such as maintenance planning (F. BahooToroody et al., 2021; Leoni et al., 2021; Leoni et al., 2020) and marine structures reliability analysis (Abaei et al., 2019; Abaei et al., 2018b). A relevant example of HBM application for reliability assessment is the methodology developed by Abaei et al. (2021). The authors adopt a multinomial process tree to model failure behaviour, while a HBM is employed to predict the failure rate of machinery inside an autonomous ship.

Within the process of estimating equipment reliability, the adoption of distinct estimation approaches could produce different results, affecting the subsequent maintenance plan. Despite all the ongoing efforts to improve the calculation of failure probabilities, there is still space to compare the application of different methodologies, especially in case a Weibull distribution is chosen to model the failure behavior. Thus, this paper aims at comparing three statistical tools to point out the most accurate in predicting the failure parameters of a given device. The methods are tested on five samples with different size, assuming a Weibull failure model.

The remainder of the paper is organized as follows; section 2 describes the steps of the proposed study along with the adopted methods. Section 3 illustrates the results arising from the implementation of the approaches to the case study, while section 4 provides the discussion of the results. At last, conclusions are presented in section 5.

1.1 Hierarchical Bayesian Modelling

The majority of statistical inference starts with 'Data', which are defined as the observations arising from a stochastic process. The process of manipulating, evaluating and organizing 'Data' leads to 'Information', while gathering 'Information' results in obtaining 'Knowledge'. At last, statistical inference is defined as the process of drawing conclusions based on what is known (D. L. Kelly & Smith, 2009). HBM is an advanced statistical tool that performs inference through the Bayes Theorem (El-Gheriani et al., 2017), showed by Equation 1.

$$\pi_1(\theta|x) = \frac{f(x|\theta)\pi_0(\theta)}{\int_\theta f(x|\theta)\pi_0(\theta)d\theta} \qquad (1)$$

where θ identifies the unknown parameters that must be estimated by the Bayesian inference. $\pi_1(\theta|x)$ represents the posterior distribution which is obtained through the multiplication of the likelihood function and the prior distribution, which are denoted by $f(x|\theta)$ and $\pi_0(\theta)$ respectively. The likelihood function describes the model from which the data have been generated and it can be interpreted as the conditional probability of obtaining the data for all admissible value of θ. On the other hand, the prior distribution represents the available information before observing the data. At last, the posterior distribution is the updated knowledge after data have been observed.

The HBM is so-named due to the exploitation of a multi-stage or hierarchical prior (D. Kelly & Smith, 2011), as illustrated by Eq. 2.

$$\pi_0(\theta) = \int_\varnothing \pi_1(\theta|\varphi)\pi_2(\varphi)d\varphi \qquad (2)$$

where φ is a vector whose components are called hyper-parameters. Prior users' beliefs or information are inserted into the analysis through the adoption of specific hyper-parameters. Finally, $\pi_1(\theta|\varphi)$ denotes the first-stage prior, which accounts the variability of φ for a certain value of θ, while $\pi_2(\varphi)$ is referred as the hyper-prior distribution, which considers the uncertainty on the vector of hyper-parameters (i.e., φ).

1.2 Maximum Likelihood Estimation

Let $Y=(Y_1,Y_2,..Y_n)$ be a random sample of observations, the MLE aims at estimating the parameters characterizing the distribution from which the aforementioned observations are most likely to have been produced. Given $\theta=(\theta_1,\theta_2,..\theta_n)$ a vector of parameters, the MLE computes the unknown parameters of interest by maximising the maximum likelihood function, showed by Equation 3 (A. BahooToroody et al., 2020).

$$f(\theta_1,\theta_2,\dots\theta_n|y) = f_1(\theta_1|y)f_2(\theta_2|y)\dots f_n(\theta_n|y) \qquad (3)$$

1.3 Least Square Estimation

Given $Y=(Y_1,Y_2,..Y_n)$ a random sample of observations and $\theta=(\theta_1,\theta_2,..\theta_n)$ a vector of parameters, the LSE aims at identifying the parameters that best fit the observed sample. Such parameters are estimated by minimizing the Sum of Squares Error, which is illustrated by Equation 4.

$$SSE(\theta) = \sum_{i=1}^{n}(Y_i - prd_i(\theta))^2 \qquad (4)$$

where Y_i denotes the i^{th} observation, while $prd_i(\theta)$ represents the prediction of the model associated to the i^{th} observation.

2.Developed methodology: materials and methods

The steps of the proposed framework are illustrated by Figure 1.

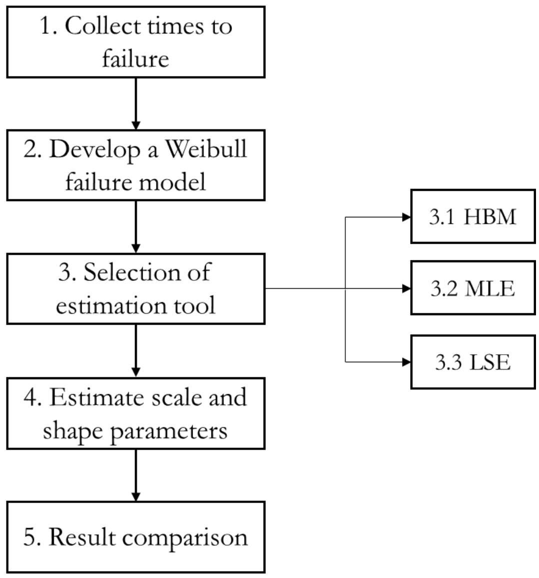

Figure 1: Flowchart of the proposed methodology.

At first, Times To Failure (TTF) are collected (step 1), then a Weibull distribution is chosen to model the failure behaviour (step 2). The Weibull failure modelling is adopted to consider the time-dependent failure rate, which is a common feature of several equipment. Subsequently, the desired estimation approach is selected (step 3), and the statistical inference is conducted to compute the unknown parameters characterizing the TTF's distribution (step 4). Finally, the results arising from the distinct methods are compared to point out the most accurate statistical tool (step 5).

2.1 Gamma-Weibull Hierarchical Bayesian Modelling

Under the assumption of a Weibull distribution for the TTFs, the Probability Density Function (PDF) is given by Equation 5.

$$f(t) = \frac{\alpha}{\beta}(\frac{t}{\beta})^{\alpha-1}\exp([-\frac{t}{\beta}])^{\alpha} \quad (5)$$

where α and β represents the shape and the scale parameter respectively. Therefore, the likelihood function of the developed HBM is expressed by Equation 6.

$$f(T_i|\alpha,\beta) = \frac{\alpha}{\beta}(\frac{T_i}{\beta})^{\alpha-1}\exp([-\frac{T_i}{\beta}])^{\alpha} \quad (6)$$

where T_i denotes the i^{th} observed TTF. The model was implemented through OpenBugs, which adopts a different parametrization for the Weibull distribution. Indeed, in this parametrization the scale parameter (λ) is obtained through Equation 7.

$$\lambda = \beta^{-\alpha} \quad (7)$$

In this study, a diffuse gamma prior is adopted for both the shape and the scale parameter (see Equation 8 and 9), as suggested by D. Kelly and Smith (2011).

$$\alpha \sim Gamma(0.0001, 0.0001) \quad (8)$$

$$\lambda \sim Gamma(0.0001, 0.0001) \quad (9)$$

The proposed Bayesian Network (BN) to model the TTFs is showed by Figure 2, where k_1 and θ_1 identifies the hyper-parameters of the prior distribution for the shape parameter, while k_2 and θ_2 are the hyper-parameters of the gamma prior characterizing the scale parameter.

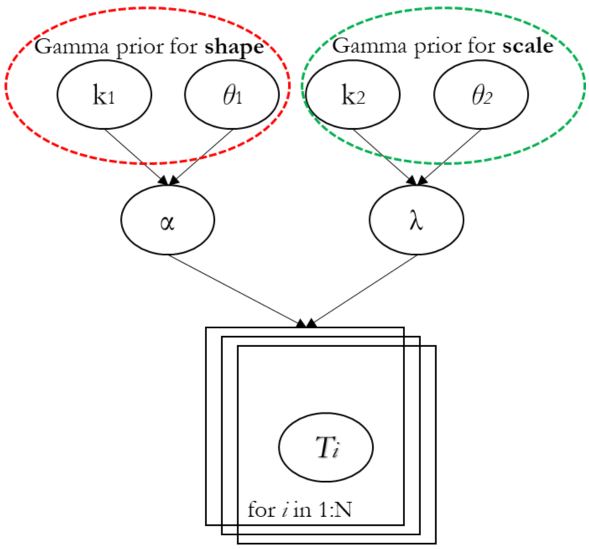

Figure 2: Adopted BN to determine the TTF's distribution.

After specifying the hyper-parameters and inserting the available data into the software, a series of Markov Chains are developed to estimate the posterior distributions of the Weibull's parameters (i.e., α, β and λ). Subsequently, the posterior mean values are extracted.

2.2 Weibull Maximum Likelihood Estimation

Given a sample $T=(T_1,T_2,..T_n)$ of TTFs, assuming a Weibull distribution, the likelihood function is expressed by Equation 10.

$$L(\alpha,\beta) = \prod_{i=1}^{n}\frac{\alpha}{\beta}(\frac{T_i}{\beta})^{\alpha-1}\exp([-\frac{T_i}{\beta}])^{\alpha} \quad (10)$$

The likelihood function is maximized numerically after setting equal to zero the logarithm of the partial derivatives with respect to a and β.

2.3 Weibull Least Square Estimation

The Weibull Cumulative Density Function (CDF) is showed by Equation 11.

$$F(t) = 1 - \exp[(\frac{t}{\beta})^{\alpha}] \quad (11)$$

To implement the LSE, Equation 11 must be rewritten as a line of the form: $y = ax + b$. By applying simple analytic steps, Equation 12 is obtained from Equation 11.

$$ln(t) = -\frac{1}{\alpha}\ln\{\ln[1-F(t)]\} + \ln(\beta) \quad (12)$$

which is a line with $y = ln(t)$, $a = -1/a$, $x = ln\{ln[1-F(t)]\}$ *(Leoni et al., 2021)* and $b = ln(\beta)$. After introducing the median rank, given by Equation 13, the shape and the scale parameter of the Weibull distribution are estimated by minimising the SSE, showed in Equation 14.

$$F(T_i) = \frac{i-0.3}{n+0.4}, \quad T_i, i = 1,2\dots n \quad (T_1 < T_2 \dots < T_n) \quad (13)$$

$$SSE(\alpha,\beta) = \sum_{i=1}^{n}\{T_i + \frac{1}{\alpha}\ln\{\ln[1-F(T_i)]\} - \ln(\beta)\}^2 \quad (14)$$

where T_i is the i^{th} observed TTF.

3.Results: application of the methodologies

3.1 Case study

To demonstrate the application of the three statistical approaches, a case study of 5 samples is considered. The 5 samples are characterized by a distinct number of observations, but they arise from the same stochastic process. The samples along with their respective dimensions are listed by Table 1.

Table 1: considered samples and their sizes.

Sample	# observations
1	15
2	20
3	30
4	40
5	50

The samples are generated through a Monte Carlo Simulation (MCS) of a Weibull distribution with a shape parameter (a) and a scale parameter (β) equal to 2 and 300 (days) respectively, corresponding to a Mean Time To Failure (MTTF) of 266 days. The MTTF represents the average time between two consecutive failures.

3.2 Application of HBM

The posterior distribution is estimated for each sample through a MCMC simulation carried out via OpenBugs. Three Markov Chains with over-dispersed initial values are adopted to conduct the inference. For each chain, 300,000 iterations are performed after 10,000 burn-in iterations.

3.3 Application of MLE and LSE

Both MLE and LSE are carried out through Minitab, and the estimated parameters of the Weibull distribution are extracted for every sample. The adopted statistical software provides also the MTTF.

3.4 First sample: 15 observations

The first sample is characterized by the least number of observations (15). The obtained results are showed by Table 2 and Figure 3.

Table 2: estimated shape parameter, scale parameter and MTTF for the first sample.

Parameter	HBM	MLE	LSE
α	1.6	1.6	1.4
β	280	272	273
MTTF	251	243	248

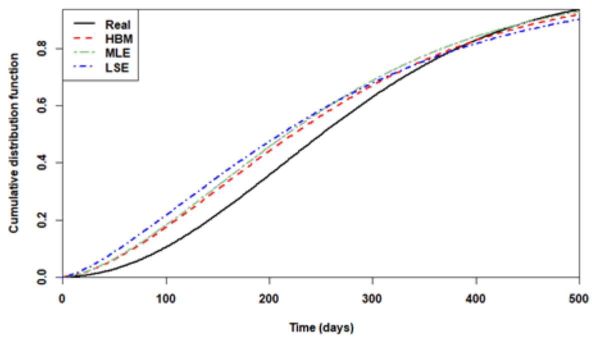

Figure 3: developed CDFs for the first sample.

The calculation revealed a greater accuracy for the Bayesian inference compared to the other approaches. Indeed, the HBM yields a posterior MTTF equal to 251 days, while the MLE and LSE compute an average time between two subsequent failures of 243 and 248 days respectively.

3.5 Second sample: 20 observations

The three approaches are implemented for the second sample as well. The estimated parameters are listed by Table 3, while the developed CDFs are illustrated by Figure 4.

Table 3: estimated shape parameters, scale parameter and MTTF for the second sample.

Parameter	HBM	MLE	LSE
α	1.6	1.6	1.4
β	266	260	258
MTTF	238	233	234

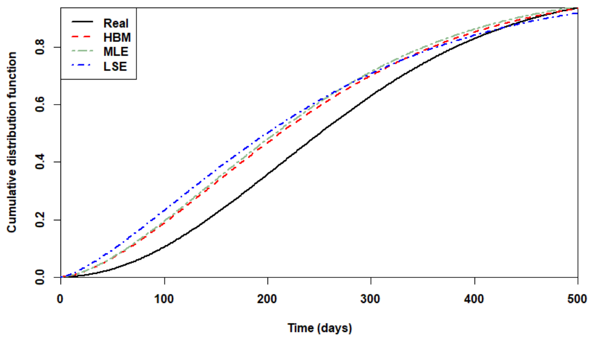

Figure 4: developed CDFs for the second sample.

The HBM showed once again a better accuracy, with a posterior MTTF of 238 days, which is 28 days shorter than the real value. On the other side, the application of the MLE and the LSE results in an estimation error equal to 33 and 32 days respectively.

3.6 Third sample: 30 observations

The third sample has 30 observations. The application of HBM, MLE and LSE depicted the results illustrated by Table 4 and Figure 5.

Table 4: estimated shape parameters, scale parameter and MTTF for the third sample.

Parameter	HBM	MLE	LSE
α	1.7	1.7	1.5
β	317	312	312
MTTF	283	278	281

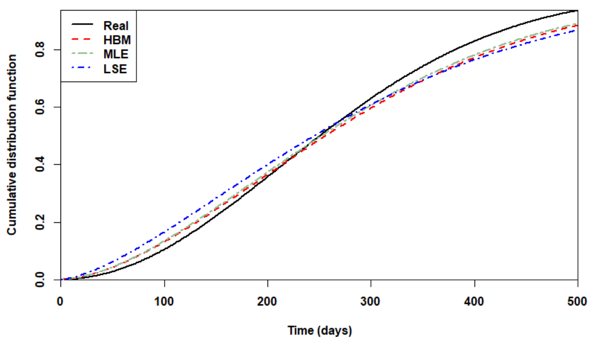

Figure 5: developed CDFs for the third sample.

The results belonging to the third sample highlighted a greater accuracy of the MLE and the LSE compared to the Bayesian inference. Indeed, the HBM provides a posterior MTTF of 283 days, while the ML and LS of MTTF are estimated at 278 and 281 days respectively.

3.7 Fourth sample: 40 observations

All the three methods are replicated also for the fourth sample, which is composed by 40 observations. Table 5 and Figure 6 reports the obtained results related to the fourth sample.

Table 5: estimated shape parameters, scale parameter and MTTF for the fourth sample.

Parameter	HBM	MLE	LSE
α	1.8	1.8	1.7
β	296	293	291
MTTF	263	260	261

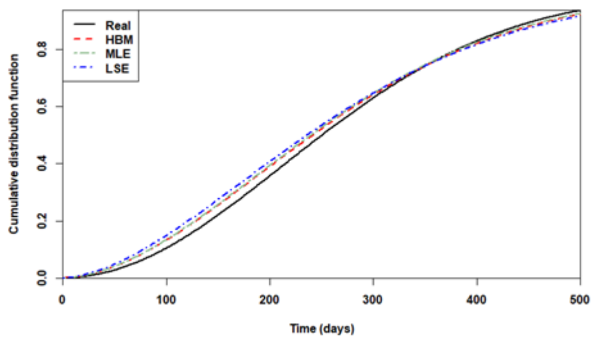

Figure 6: developed CDFs for the fourth sample.

As highlighted by Figure 6, the three approaches produce similar results. Moreover, the accuracy of the prediction increases compared to the previous applications. The greater accuracy is related to the higher number of observations. The MTTF is estimated at 262 days by the HBM, while MLE and LSE predicts an average time between two subsequent failures of 260 and 261 days respectively.

3.8 Fifth sample: 50 observations

Finally, the results of the fifth sample are reported by Table 6 and Figure 7.

Table 6: estimated shape parameters, scale parameter and MTTF for the fifth sample.

Parameter	HBM	MLE	LSE
α	1.9	1.9	1.8
β	307	305	305
MTTF	272	272	270

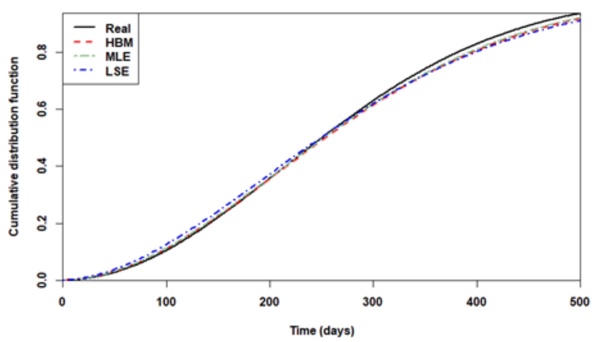

Figure 7: developed CDFs for the fifth sample.

As illustrated by Figure 7 almost no difference is seen among the three methodologies. The predictions of HBM, MLE and LSE are very similar. Indeed, the HBM yields a posterior mean of 272 days, while both the MLE and the LSE provides a MTTF equal to 271 days.

4.Discussion: comparison of the methodologies

To make a meaningful difference and compare the three approaches, the estimated MTTFs are converted into dimensionless value through the real MTTF (i.e., the one adopted for the MC simulation). The results arising from the calculation are showed by Table 7 and Figure 8.

Table 7: dimensionless MTTF for each sample and each approach.

Sample	HBM	MLE	LSE
1	0.944	0.914	0.932
2	0.895	0.876	0.880
3	1.064	1.045	1.056
4	0.989	0.977	0.981
5	1.023	1.015	1.023

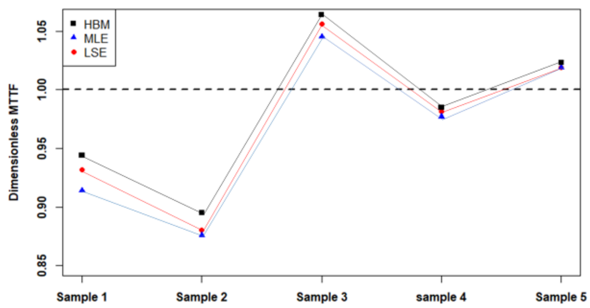

Figure 8: dot-plot of the dimensionless MTTF for each sample and each approach. The dotted horizontal line represents the real MTTF.

For three samples out of five the HBM denotes a higher accuracy. By contrast, the application of MLE and LSE produces smaller estimation errors for the third and the fifth sample. Furthermore, the HBM emerged as the most accurate approach for the sample characterized by few observations, while increasing the number of observations leads to obtaining similar predictions from all the estimation tools. It is worthwhile mentioning that for smaller samples the estimations manifest poor accuracy, while increasing the sample size results in obtaining more accurate results.

To determine the method characterized by the higher accuracy, the Root Mean Square Error (RMSE), defined by Equation 15, is calculated.

$$RMSE_j = \sqrt{\frac{\sum_{i=1}^{n}(MTTF_{i,j} - MTTF_{REAL})^2}{n}} \qquad (15)$$

where $MTTF_{i,j}$ identifies the average time between two consecutive failures estimated by the j^{th} approach for the i^{th} sample, while $MTTF_{REAL}$ and n denote the real average time between the consecutive failures and the considered number of samples respectively. The RMSE computed for the HBM, the MLE and the LSE are respectively equal to 269, 363 and 327 days respectively. Consequently, the adoption of HBM will generally result in more accurate probabilities of failure, leading to a much safer and cost-effective maintenance plan.

5.Conclusions

This paper presents the comparison of three estimation approaches within the process of reliability analysis. The application of the methodologies was implemented on five samples with distinct dimensions, all of which arise from the same Weibull distribution. The HBM emerged as the most accurate statistical tool since it has showed the lowest RMSE within distinct samples. Moreover, while for greater sample sizes the application of the three approaches determined similar results, for smaller samples the HBM is characterized by the lowest error. Considering the aforementioned statements, it is strongly recommended to adopt Bayesian inference, which will provide a more efficient maintenance plan, without overlooking safety aspects. However, to prove the advantages of HBM over MLE and LSE, some tests with more samples and distinct distributions are required. Moreover, the convergence analysis of the iterations based on the starting points must be addressed. For this study, no prior information was incorporated into the Bayesian analysis. Inserting some weakly prior beliefs into the estimation process could increase the accuracy for the sample with few observations. Therefore, further developments could include into the HBM some valid prior knowledge to predict the failure parameters.

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Appendix A. OpenBugs' code

The script implemented in OpenBugs to perform the Bayesian inference is showed by Figure 9. The code can be found in (D. Kelly & Smith, 2011) and (D. L. Kelly & Smith, 2009).

```
model {
      for(i in 1:N) {                          #Consider all elements in the sample

            time[i]~dweib(alpha, lambda)       #Weibull likelihood function

                  }

      alpha~dgamma(0.0001,0.0001)              #Gamma prior for the shape parameter

      lambda~dgamma(0.0001,0.0001)             #Gamma prior for the scale parameter

      beta<-pow(lambda,-1/alpha)               #Common adopted scale parameter

}
```

Figure 9: OpenBugs' script for the Gamma-Weibull failure modelling.