Digital Twin-Based Predictive Production Control

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Abstract: Cyber-physical systems and the fourth industrial revolution are reshaping the way production systems are modeled and controlled. Since their emergence as new simulation paradigm, Digital Twins (DT) have been supporting decisions in various industrial engineering fields, and production planning and control is a significant scope for future advances with the aid of this technology. Literature reports interesting works on DT-based production scheduling. However, only few research works related to DT-based production control were found in the literature and in particular a literature gap lies in investigating the use of DT for enhancing existing production control mechanisms, despite it is a promising technological support for granting robustness and stability to the manufacturing systems performance. The authors aim to contribute to this topic by proposing a new framework for model-based production control grounded on DT simulations, focusing on WIP and throughput control. Thanks to the predictive capabilities provided by the DT, an iterative optimization of production control parameters over a rolling horizon has been developed, also inspired by the theory on model-based predictive control. The framework includes load-oriented centralized and decentralized control procedures, improved by using a Particle Swarm Optimization (PSO) aimed to recursively set the WIP limits according to estimates of production outputs. The proposed framework has been validated through simulation in a general flow shop case, proving that the framework is effective in increasing throughput while reducing WIP.

Keywords: Digital Twin, simulation, production control, model-based control

I. INTRODUCTION

The digitalization of manufacturing represents a crucial transformation for today's industry and a great opportunity to achieve higher levels of productivity while reducing costs.

The advent of new digital capabilities within Industry 4.0 stream has transformed several managerial functions. Production Planning & Control (PPC) could be innovated to enable the creation of further value through the usage of new technologies [1].

Among the Industry 4.0 digital technologies, Cyber-Physical Systems can open the way to real-time monitoring and synchronization of the shop floor activities to the virtual space [2]. In fact, the cyber part can realize computations that include data analysis and simulations. Within this context, the concept of Digital Twin (DT) has emerged as the new paradigm for simulation synchronized with the field [3].

Implementing DT within Industry 4.0-enabled manufacturing systems could allow exploiting their capabilities to optimize production [4]. Exploiting DT for supporting production control still represents an almost unexplored field in research, especially in the concern of order release methods.

The rest of the document is organized as follows. Section II reviews the relevant literature, and section III defines the objectives of this research work. The proposed framework is presented in section IV and some details

related to its implementation are provided in section V. The experimental results are reported in section VI, and they are discussed in section VII; finally, section VIII includes conclusions and future research directions.

II. REVIEW OF THE LITERATURE

This research work originates from an analysis of the existing literature concerning the use of DT for control in manufacturing, particularly focusing on methods for releasing orders.

Several research works deepen this concept emphasizing the simulation aspect of the DT as a core component of various optimization frameworks. Instead, other works consider the DT model as purely data-driven, often with a black-box approach.

Cimino et al. present a DT based on a Discrete Event Simulation (DES) model in order to improve the visibility of production events, which could be useful for production control [5]. Mykoniatis et al. proposed a DT based on a hybrid model able to combine the advantages of DES and Agent-Based Modelling [6]. Ragazzini et al. present a DT based on a DES model which performs a what-if analysis to allow a reinforcement learning agent to adjust and, thus, improve the parameter of CONWIP [7].

Park et al. illustrate how DTs play a role in providing the virtual environment for training machine learning algorithms [8], [9]. The work presented by Min et al. represents a relevant example of how the machine

learning-based production control can find the optimized real-time control parameters based on the DT model and real-time data [10]. Donhauser et al. considered production control based on a predictive-reactive model to analyse the impact of the deviations from a baseline [11]. May et al. present the situational control agent selection as an additional feature of DT that could improve production control [12]. In fact, it allows to select the most suitable control policy for the current condition, based on the execution of multiple DT simulations. The works proposed by Guo et al. represent two relevant examples in this sense in which the DT integrates and synchronizes production jobs data between physical and cloud space for controlling operations [13], [14]. Also, a real-time ticket system inspired by Kanban control theory is presented. Similarly, Zhou et al. worked on another proposal for the digitalization of Kanban with the aid of DTs [15]. The authors describe a way to produce and manage order data in real-time through the knowledge-based intelligent skills of the DT. Both Borangiu et al. and Ma et al. deepen the analysis of artificial intelligence and DT, presenting DT based on predictive models [16], [17]. The predictive models built using neural networks, can both assess the current state and evaluate future ones. The models are trained with historical and sample data, but predictions are performed according to the data acquired in real-time.

Overall, the literature findings provide evidence that DT are helpful for the decision-making within production monitoring and control through real-time simulation, improving or optimizing the manufacturing systems performances, when used in addition to control methods. Nevertheless, the critical analysis of the same findings also allows to identify a relevant gap in the scientific literature. In fact, to the best of the authors' knowledge, there are no research studies aiming at using DT to innovate and improve existing production control mechanisms. Furthermore, there is no tangible evidence of how Industry 4.0-enabled smart capabilities may support the decision-making process of industrial control during the shop floor operations through simulation.

III. RESEARCH DESIGN

According to the results of the literature review and the identified knowledge gaps, the main objective of this work is formulated: this work aims at the development of a DT-based optimization framework for improving an existing order release mechanisms through online adjustment of its parameters. This is granted by the DT predictive capabilities that are utilized for the support of near real-time decision-making processes in production control.

In order to achieve this objective, a simulation-based optimization framework is proposed including some elements of Model Predictive Control (MPC) theory [18].

The similarities between the DT and MPC lie in the use of a model representing the current state of the physical system and in the possibility to use such a predictive model to iteratively optimize the control of the physical system. Indeed, the similarities between the "sensor-tocontroller" and "controller-to-actuator" (typical of advanced control systems) and the physical-to-virtual and the virtual-to-physical world (characteristics of the DT) were previously identified in a review by Jones et al. [19]. Moreover, this would be aligned also with the definition of DT provided by Kritzinger et al., since it could allow closing the loop thanks to the feedback provided to the physical system, enabling autonomous control actions [20].

An existing production control mechanism including both a centralized and decentralized part could be selected as a starting point for this work. Indeed, this would allow the developed framework to have a broader impact on the system [21], [22].

IV. PROPOSED FRAMEWORK

A novel optimization framework is proposed to enable the adoption of the DT for improving a traditional production control mechanism through the online update of its control parameters. The proposed framework includes four key elements:

- the physical production system (or even, in a prototypical case, also a simulation model replicating its connectivity capabilities; in this specific case, we may talk of a physical twin);
- 2. the DT of the considered production system;
- 3. the production control system, applied both on the physical system and on its DT;
- 4. the optimization algorithm, working within the virtual space and interconnected with the DT to perform simulation-based optimization.

A detailed description of the proposed framework is discussed hereafter, while its schematic representation is provided in Fig. 1.



Fig. 1. Proposed framework

A. Digital Twin

The DT is based on a DES model, and it is synchronized with the physical system to mirror its current state. The

information required by the DT includes the Work In Progress (WIP) of each workstation and the state of each workstation. In other words, the DT also contains the state of each order released on the shop floor.

The current system state is the starting point for the prediction of the future performances through the simulation capabilities of the DT. The results of the simulations performed by the DT will be crucial for the optimization of system performances in near real time.

B. Production control

The production control system considered for this work is adapted from DEWIP protocol proposed by Lödding [23]. It has been selected because it is constituted of both a centralized and a decentralized part. Despite this, the only control variables to be considered are the WIP limits, one per workstation in the considered system. The WIP limit represents the maximum WIP allowed on a certain workstation.

- Centralized control refers to the orders release mechanism. Prior to releasing an order for production, the WIP in the first and the second workstations in the order routing is checked to verify the compliance with their WIP limits. Thus, orders can be released only when the WIP on the first two workstations fall below their WIP limits;
- Decentralized control refers to the control of the orders already released on the shop floor. An order can proceed to the next workstation in its routing only if WIP limits are not exceeded. Indeed, for each completed order on any workstation, the following workstation in the order routing determines whether to authorize the arrival of the new order according to its current WIP and its WIP limit. Indeed, if the WIP on the workstation exceeds the workstation WIP limit the authorization is refused. Otherwise, the order can proceed, and it is moved to the workstation.

C. Optimization algorithm

To perform the optimization a Particle Swarm Optimization (PSO) algorithm is adopted and coupled with the DT. The optimization is based on the concept of rolling horizon, borrowed from the MPC theory. The reduced computational time required to obtain a solution and possibility to reuse the whole population of suboptimal solutions suggested the choice of this metaheuristic algorithm. Other optimization algorithms could be used in the framework and their comparison will be object of a future study.

The optimization is performed repeatedly with a predefined frequency, in a time-based fashion, thus resulting in the split of the time horizon in a series of control steps.

At the end of each control step, the optimized WIP limits obtained are fed back to the physical system, enabling their dynamic adjustment. Indeed, the physical system uses the optimized WIP limits received at the previous control step to control the order release and the operations of the orders already released in production in the physical system. The optimization process is depicted in Fig. 2.

Given the characteristics of the selected production control method, both decentralized and centralized control can be improved by the DT by dynamically acting to adjust the WIP limits.



Fig. 2. Sequence diagram of the optimization process

A novel objective function is also proposed, in order to drive the DT-based optimization process. The objective function is composed of two parts to account both for the WIP and for the throughput (TH), aiming at minimizing WIP and maximizing TH. The first expression indirectly considers the WIP by accounting for the sum of the ratios between of WIP limit and the average WIP limit in correspondence with the maximum TH. The second term represents the ratio between the TH and the maximum TH on the line under the selected production control protocol. Weight parameter α must be set through the analysis of multiple experiments.

Each solution considered in the optimization process includes a value of WIP limit for each workstation of the considered production system to be optimized. A random variation of the WIP limits is fed to the DT to evaluate its effect on the future throughput of the system. The PSO interacts again with the DT by assessing the effect of the predicted system's throughput in the expression of the objective function to be minimized, whose aim is to reduce the WIP limits while taking care of the detrimental effect that this reduction has on the throughput of the system.

$$\min f(TH, WIP_i) = \alpha \sum_{i=1}^{n} \frac{WIP_i}{\overline{WIP_i}} - (1 - \alpha) \frac{TH}{TH_{max}}$$

s.t. $WIP_{min} < WIP_i < WIP_{max}$

where:

$$\alpha$$
 weight

 $WIP_i = WIP$ limit of workstation *i*

TH = throughput

 TH_{max} = maximum throughput of the system

 \overline{WIP}_i = mean WIP limit of workstation *i*

in correspondence of TH_{max}

 WIP_{min} = lower bound for WIP limits

 WIP_{max} = upper bound for WIP limits

V. IMPLEMENTATION

For the validation, the methodology proposed by Barbieri et al. was adopted [24]. It describes how to implement a DT-based scheduling framework connecting the DT to a virtual commissioning model rather than to the system itself. Similarly, also Ait-alla et al. defined a methodology to evaluate a DT without establishing a connection with the real system, using its so-called *physical twin* instead [25]. Thus, the physical production system is replaced with its model, based on DES.

The type of production system selected for the purpose of this work on which the experiments were performed is the general flow shop. It is thoroughly described in the work by Oosterman et al. and also depicted in Fig. 3 [26]. Most relevant parameters were taken from the work by Thürer et al. and are reported in Table I [27]. The main difference with respect to the well-known pure flow shop configuration is that jobs do not have to be processed in every work center and thus they only visit the ones required from routing specifications.



Fig. 3. General flow shop [26]

TABLE I				
Parameter	Value			
No. of work centres	6			
Re-entrant flows	No			
Routing length of jobs (operations per job)	Discrete, uniform distribution [1, 6]			

Operation processing times	Truncated 2-Erlang distribution (mean = 1[min], max = 4 [min])
Inter-arrival times	Exponential distribution (mean = 0.648 [min])
Queue lengths	Limited and all equal (15 jobs)
Simulation time	60 [min]

VI. EXPERIMENTAL RESULTS

The proposed model has been assessed through computational experiments to quantitatively evaluate the effectiveness of the proposed DT-based production control system. The performances of the system enhanced with the proposed DT-based optimization model are compared against the ones obtained with the standard DEWIP method. Within this work it is also called static control method, since the control parameters were optimized before running the experiments, but they are not changed adaptively. The improvements brought by the DT-based optimization model have been investigated following two main strategies: TH comparison (A) and WIP comparison (B).

A. TH comparison

The first analysis performed aims at studying how the proposed framework could improve the TH of the system under analysis for similar WIP average levels. For this reason, the performances obtained implementing the DTbased optimization for production control are compared to those obtained with the use of the same production control method after optimizing its parameters for maximizing productivity. To achieve this, ten runs for each experimental setup were performed and the statistical analysis of the experimental results is reported in Table II.

TABLE II Summary data						
Model Performance Mean StDev 95%						
DT- based	TH [ord/h]	86.00	5.12	(82.88; 89.12)		
	WIP [min]	82.09	8.26	(76.12; 88.06)		
Static control	TH [ord/h]	75.30	4.22	(72.18; 78.42)		
	WIP [min]	86.20	9.52	(80.68; 91.72)		

An Analysis Of Variance (ANOVA) is reported in Table III. It proves that TH is higher adopting the DT-based approach for control with statistical significance. Fig. 4

graphically represents the results in terms of TH for each experiment.

TABLE III ANOVA FOR TH					
Source	DF	Adj SS	Adj MS	F- Value	P- Value
Factor	1	572.5	572.45	26.01	0.000
Error	18	396.1	22.01		
Total	19	968.6			



Fig. 4. Interval plot for TH

A paired t-test was performed to quantify the difference in TH between the experiments. The results are reported in Table IV, while Fig. 5 shows a boxplot of the differences in WIP between the two experiments.

TABLE IV PAIRED T-TEST FOR TH					
Mean	StDev	SE Mean	95% CI for	T- Value	P- Value
10.70	4.32	1.37	(7.61; 13.79)	7.83	0.000



Fig. 5. Boxplot for TH differences

To conclude this part of the experimental analysis, the equivalence of the average WIP values in the two experiments must verified. For this reason, a Tukey test is performed and the results depicted in Fig. 6 show that the confidence interval includes zero and thus WIP values cannot be considered statistically different. This suggests that the TH increase granted by the DT-based approach for production control is achieved without any significant improvement in the average WIP.



B. WIP comparison

An additional analysis was performed to prove that the application of the DT-based optimization model leads to lower WIP while keeping a similar value for the TH of the production system. Table V reports statistical analysis for the ten runs of each experiment.

TABLE V SUMMARY DATA					
Model	Performance	Mean	StDev	95% CI	
DT- based	TH [ord/h]	86.00	5.12	(82.88; 89.12)	
	WIP [min]	82.09	8.26	(76.16; 88.01)	
Static control	TH [ord/h]	83.70	5.17	(80.28; 87.12)	
	WIP [min]	92.73	9.66	(86.76; 98.70)	

Table VI contains an ANOVA, proving statistical significance of the difference between the WIP for the two experiments, proving that WIP is lower with the DT-based approach for control. Moreover, Fig. 7 graphically represents the outcomes in terms of WIP for the two experiments.

TABLE VI ANOVA FOR WIP					
Source	DF	Adj SS	Adj MS	F- Value	P- Value
Factor	1	566.2	566.15	7.01	0.016
Error	18	1453.7	80.76		
Total	19	2019.9			



Fig. 7. Interval plot for WIP

A paired t-test was performed to quantify the difference between the experiments. The results are reported in Table VII. Fig. 8 shows a boxplot of the differences in WIP between the two experiments.

TABLE VII PAIRED T-TEST FOR WIP					
Mean StDev SE 95% CI T- P Mean for Value Va					
-10.64	5.58	1.76	(-14.63; -6.65)	-6.03	0.000



Fig. 8. Boxplot for WIP differences

Finally, a Tukey test is performed to check that the two experiments report almost the same TH values. As shown in Fig. 9, the confidence interval includes zero, and therefore they are not significantly different.



VII. DISCUSSION

The experimental results prove the effectiveness of the proposed framework in improving the TH and WIP performance of the general flow shop under analysis.

The DT-based optimization supports dynamic adjustments in the production control protocol allowing to increase flow shop TH while reducing WIP. As remarked previously, two separate analyses were conducted to analyze the performance for both performance objectives separately, since the proposed method cannot be strictly classified as multi-objective optimization .Indeed, the weighted sum method adopted reduces the problem to single-objective optimization [28]. In fact, both TH and WIP are normalized and just summed according to the weight parameter α , rather than searching for a set of optimal solutions minimizing both.

Regarding the improvements in productivity, the first analysis shows a statistically significant improvement of 14.2% (Table II). The second analysis proves that by achieving similar levels of TH, the DT-based production control achieves WIP levels that are 11.5% lower with statistical significance (Table V). This proves that for both performances considered, the DT-based optimization can provide improved performances.

VIII. CONCLUSIONS

This work proposed a novel optimization framework for production control based on DT which is a problem still little studied in literature, despite the existing promising approaches in production control. It is grounded on the principles of simulation-based optimization, according to which the algorithm obtains a solution by testing different control levels in the DT simulation environment prior to being deployed on the field. This work exploits the real-time capabilities of the DT to improve the overall system's TH while reducing the WIP. Moreover, the concept of rolling horizon for optimization is borrowed from the MPC theory.

Overall, production control decisions are supported by a framework taking decisions to dynamically limit the WIP on each station. For this reason, the DT is applied to both decentralized and centralized control by adjusting the same set of control variables.

Experimental results proved that the dynamic adjustment of the WIP limits during operations leads to better results in system performances compared to the static setting of the optimized control variables. It was shown that the framework can improve both TH and WIP. In fact, results show similar improved TH values for similar WIP and reduced WIP for comparable production output in terms of TH.

From an academic viewpoint, this work contributes to DT and to production control theories proving the possibility of enhancing the performances of production systems by acting on control methods through DT-based optimization. According to the achieved results in terms of WIP and TH, potential benefits for the industry include the possibility to better fulfill the strategic logistic objectives related to cost and performances, as they are strongly influenced by production control methods.

The main limitation of this work is related to the validation of its industrial applicability. The challenges of deploying such a complex production control protocol based on DT should be further analyzed through on-field studies.

Future works should include the improvement of the optimization stability to grant fewer changes in WIP levels, to decrease system nervousness, and to possibly accelerate the optimization process by narrowing the solution search space. Moreover, the proposed framework shall be applied considering also different production control methods. An analysis of the main factors involved in this study could be relevant, and multi-objective optimization could be considered too. Finally, the deployment of the proposed method in a real factory should be seen as the real benchmark to assess new DT-based optimization models for production control.

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