A mathematical model for choosing between centralization and decentralization in distribution networks for perishable products

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Abstract: An optimal supply chain (SC) configuration improves the performance of distribution companies for perishable products since it reduces logistic costs while ensuring high service levels and preventing product spoilage. The SC configuration should not be optimized only once, but rather continuously reviewed since customer demand for perishable products varies over time. A key decision to be reviewed is whether to centralize or decentralize inventories. This decision allows for matching the allocation of stocks in distribution centers (DCs) with customer needs, ensuring product availability, reduced waste due to product spoilage, and lower logistic costs. However, distribution companies for perishable products struggle in reviewing their SC configuration since structured methodologies for choosing between centralization and decentralization are missing. Due to this gap, the performance of distribution companies for perishable products is often undermined. To address this gap, this work provides distribution companies for perishable products with a novel mathematical model that allows comparing five SC configurations, where inventories are either centralized, decentralized, or managed with three hybrid configurations. Through the mathematical model, the optimal SC configuration is identified as the one with minimum total logistic cost, which also respects pre-established service levels and the product shelf life. The applicability of the mathematical model is tested in the case study of an agri-food distribution company. The results show that, by adopting the proposed model, distribution companies can associate individual perishable products with the optimal SC configuration, achieving cost savings, high service levels, and reduced waste. Moreover, the results confirm that decentralization is advantageous for perishable products with short shelf lives, while centralization is cost-effective in other cases.

Keywords: distribution network configuration; supply chain management; supply chain design; perishable goods.

I. INTRODUCTION AND BACKGROUND

Distribution companies that handle perishable products are responsible for procuring, storing, and distributing goods with a limited shelf life (e.g., drugs, food, and blood). These products have an expiry date after which they spoil and must be discarded [1]. To remain competitive in the global market, distribution companies for perishable products must optimize their performance by ensuring high service levels, minimizing waste due to products' spoilage, and reducing logistic costs associated with procurement, storage, and distribution [2][3]. An effective strategy to achieve this optimization is to optimally configure Supply Chains (SCs), which involves determining how many Distribution Centers (DCs) to establish, where to geographically locate them, how to serve customers with DCs, and what inventory control policies to adopt for each perishable product [4], [5]. Specifically, choosing between centralized, decentralized, and hybrid SC configurations has been reported as a crucial decision [6] in SC configuration. A centralized SC configuration entails storing inventories of products in a single DC that serves all customers. This SC configuration reduces holding and ordering costs to replenish DCs due to the 'risk-pooling' effect [7]. Moreover, centralization reduces the waste cost of spoiled products due to high inventory turnover. However, it leads to high transportation costs and reduced SC responsiveness due to the average distance between the central DC and customers [8]. Conversely, a decentralized SC configuration involves storing inventories in multiple DCs, each serving a local customer. This SC configuration offers benefits in terms of SC responsiveness and reduced transportation costs. However, it leads to higher holding, ordering, and waste costs since many DCs are managed, each containing several stocks [9]. Finally, a hybrid SC configuration strike a balance by choosing an intermediate number of DCs between centralization and decentralization, thereby obtaining intermediate benefits of SC responsiveness and total logistic distribution costs [10]. Although optimizing SC configurations is crucial, choosing between centralized, decentralized, and hybrid SC configurations poses three main challenges. First, the optimal alternative between centralized, decentralized, and hybrid SC configurations must be identified as the one that mitigates conflicting needs such as lowering inventory levels while ensuring products' availability. This involves seeking a trade-off among multiple cost items with different behaviors, comprising the cost of purchasing stocks, the ordering costs, the holding costs of inventories, the transportation costs for distributing products, backorder costs, and the waste costs associated with product spoilage [11]. The second challenge is the identification of the optimal alternative between centralized, decentralized, and hybrid SC configurations for each perishable product, which can be daunting considering that distribution companies usually manage thousands of different products. Finally, concerning the last challenge, the SC configuration should not be optimized once, but periodically reviewed since customer demand for perishable products typically varies over time [12]. To overcome the aforementioned challenges, distribution companies for perishable products should adopt structured methodologies to optimize their SC configuration, reviewing the choice of centralized, decentralized, and hybrid SC configurations over the years. However, as recently claimed [13], the scientific literature lacks these structured methodologies, jeopardizing the companies' ability to optimize their performance. Upon further examination of the extant literature, three main gaps emerge. First, few research have been performed to compare centralized, decentralized, and hybrid SC configurations, with most of existing studies focusing on qualitative comparisons [14]. Among the existing quantitative studies, the majority propose mathematical programming models to optimize the SC configuration of specific distribution networks for perishable products, but examining the results solely from a computational perspective (focusing on how to solve certain NP-hard problems and indicating the time, cores, and number of iterations required to achieve an optimal solution) [15]. Conversely, there is a lack of quantitative studies that compare the operational performance of different SC configurations in general industrial contexts. This absence leaves distribution companies for perishable products without guidance on how to choose between centralized, decentralized, and hybrid SC configurations [16]. Regarding the second literature gap, existing studies have conducted cost-benefit analyses of specific SC configurations, but they have only considered fixed costs of DCs along with holding and transportation costs [4]. However, other crucial cost items like ordering, backorder, and waste costs of spoiled products have been overlooked even if they impact the total logistic distribution cost [17] [18]. Lastly, as for the last literature gap, the problem of reviewing the SC configuration over time has rarely been addressed in the literature [17], which prevents distribution companies for perishable products from adapting centralization or decentralization choices to the ever-changing customer demand. Aiming to fill the three aforementioned gaps, this work proposes a novel mathematical model to assist distribution companies of perishable products in reviewing their SC configuration. The proposed mathematical model quantitatively compares five SC configurations: a centralized, a decentralized, and three hybrid SC configurations. The comparison is based on the economic evaluation of their respective total logistic distribution cost, including in the investigation the following cost items: holding, ordering, transportation, and backorder costs, and the waste costs of spoiled products. The optimal SC configuration is selected as the one with minimum total logistic distribution cost, which also respects pre-established service levels and products' shelf life. The remainder of this paper is as follows. Section 2 introduces the problem addressed by the mathematical model and the simplifying assumptions. Section 3 presents the mathematical model. Section 4 provides a case study to tests the mathematical model. Finally, Section 5 offers conclusion remarks.

II. PROBLEM AND ASSUMPTIONS

The proposed mathematical model aims to determine the optimal SC configuration for two-echelon distribution companies handling perishable products. Specifically, this model provides recommendations for each perishable product, suggesting the optimal alternative to be adopted among a centralized, a decentralized, and three hybrid SC configurations ($i = 1,2, \dots 5$ in Figure 1). These five alternatives have been selected based on [18] and varying the degree of centralization (Deg_i) as in Equation 1. Deg_i discerns different SC configurations based on the number of DCs (DC_i) able to fulfill the customer demand and the number of customers to be served (N). Accordingly, Deg_i ranges between 0 and 1, where 0 represents decentralization, 1 is centralization, and 0.25, 0.50, and 0.75 are hybrid SC configurations. Among the five SC configurations, the mathematical model selects the optimal alternative by seeking the one associated with the minimum total logistic distribution cost (i.e., the sum of cost items in Table I), which also respects a pre-established service level and the product’s shelf life.

$$Deg_i = \begin{cases} 1, if\ i = 5\ (centralization) \\ 1 - \frac{DC_i}{N}, else \end{cases} \qquad (1)$$

The mathematical model was developed based on several simplifying assumptions: (i) the considered perishable product has a deterministic shelf life [1]; (ii) customer demand follows a normal distribution [10]; (iii) the procurement lead time is deterministic, depending on the perishable product, not on the geographical location of DCs [19]; (iv) the procurement lead time is shorter than the product’s shelf life [20]; (v) no capacity constraints are considered in DCs [18]; (vi) establishing a certain SC configuration in Figure 1, all DCs have the same average transportation cost [18]; (vii) a continuous (RP,Q) inventory policy is used to control stocks in DCs, where RP is the reorder point and Q is the optimal order quantity [10]; (viii) stocks are withdrawn from DCs based on a First-In-First-Out policy [21]; (ix) no lateral transshipments are allowed [1]; (x) the transport vehicle used for product distribution is the same in all SC configurations, with a capacity constraint on the maximum number of products that can be transported per trip. Moreover, its unitary transportation cost is a constant per kilometer [1]; (xi) the time horizon considered to develop the analysis is one year [18]; (xii) fixed costs of DCs are neglected since, when reviewing an existing SC configuration, companies already own DCs.

TABLE I
NOMENCLATURE OF THE MATHEMATICAL MODEL

Index	Description	Unit measure
i	SC configuration. i=1, 2, …, 5	-
Input parameter	**Description**	**Unit measure**
N	Number of customers served	-
Deg_i	Degree of centralization (can be 0, 0.25,0.50, 0.75, or 1)	-
SL	Expected service level. It is associated with the service factor z in a standard normal distribution	-
m	Product’s shelf life	time
$\bar{D}$	Mean demand of one customer for the perishable product	units/time
σ	Standard deviation of the annual demand emitted by one customer for the product	units/time
b	Unitary backorder cost of the perishable product	€/backorder
L	Procurement lead time of the perishable product	time
c	Unitary cost of purchasing the perishable product	€/unit
o	Cost of issuing a supply order	€/order
h	Inventory holding cost rate	time^{-1}
w	Unitary waste cost of a spoiled product	€/unit
d_c	Average distance from central DC to customers (when $i = 5$)	km
t	Unitary transportation cost	€/km*vehicle
v	Average speed of vehicles	km/time
q	Average quantity of perishable product ordered by a customer each time it demands it	units/demand
C	Capacity of transport vehicle	units
Decision variable	**Description**	**Unit measure**
Q_i	Optimal reorder quantity of the perishable product in a DC	units
RP_i	Reorder point associated with the perishable product in a DC	units
SS_i	Safety stocks of the perishable product in a DC	units
DC_i	Number of DCs in the SC	-
D_i	Annual demand received in a DC for the perishable product	units/time
I_i	Average inventory level of the perishable product in a DC	units
N_v	Average number of vehicles to transport the perishable product	vehicles/transportation
N_{t_i}	Average number of transports to distribute the perishable product	transportations/time
d_i	Average distance from a DC to customers	km
tt_i	Average time required to transport the product	time
ts_i	Longest time spent by the perishable product in a DC	time
N_{w_i}	Average number of spoiled products for each reorder lot	units/time

N_{b_i}	Average number of backorders for the product	orders/time
N_{o_i}	Average number of supply orders for the product	orders/time
Evaluated cost	Description	Unit measure
C_{TOT_i}	Total logistic distribution cost	€/time
C_{P_i}	Annual purchase cost	€/time
C_{H_i}	Annual holding cost	€/time
C_{O_i}	Annual ordering cost	€/time
C_{B_i}	Annual backorder cost	€/time
C_{T_i}	Annual transportation cost	€/time
C_{W_i}	Annual waste cost	€/time

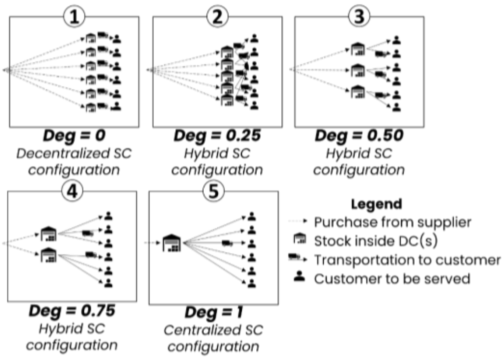

Figure 1. Five investigated SC configurations

III. MATHEMATICAL MODEL

The proposed mathematical model is a mixed integer linear programming model that relies on the notation presented in Table I. For each perishable product managed by the distribution company, the mathematical model targets the identification of the SC configuration (i) with the minimum total logistic distribution cost (C_{TOT_i}) according to Equations 2-3. The cost items in Equation 3 are described in Table 1 and they can be calculated based on Equations 4-9, which in turn depend on Equations 10-19. Notably, Equation 17 is based on [18], while Equations 18-19 depend on the inventory control policy adopted for the perishable product, as explained in Appendix A.

$$min[C_{TOT_i}] \; for \; i = 1,2,\dots,5 \quad (2)$$

$$C_{TOT_i} = C_{P_i} + C_{H_i} + C_{O_i} + C_{B_i} + C_{T_i} + C_{W_i} \quad (3)$$

$$C_{P_i} = c \cdot D_i \cdot DC_i \quad (4)$$

$$C_{H_i} = h \cdot c \cdot I_i \cdot DC_i \quad (5)$$

$$C_{O_i} = o \cdot N_{o_i} \cdot DC_i \quad (6)$$

$$C_{B_i} = b \cdot N_{b_i} \cdot DC_i \quad (7)$$

$$C_{T_i} = t \cdot N_v \cdot N_{t_i} \cdot d_i \cdot DC_i \quad (8)$$

$$C_{W_i} = w \cdot N_{w_i} \cdot N_{o_i} \cdot DC_i \quad (9)$$

$$D_i = \frac{\bar{D} \cdot N}{DC_i} \quad (10)$$

$$DC_i = \begin{cases} 1, if \;\; i = 5 \\ [(1 - Deg_i) \cdot N]^+, \; else \end{cases} \quad (11)$$

$$I_i = \frac{Q_i^*}{2} + SS_i \quad (12)$$

$$N_{o_i} = \frac{D_i}{Q_i^*} \quad (13)$$

$$N_{b_i} = (1 - SL) \cdot D_i \quad (14)$$

$$N_v = \left[\frac{q}{C}\right]^+ \quad (15)$$

$$N_{t_i} = \frac{D_i}{q} \quad (16)$$

$$d_i = \begin{cases} d_c, if \;\; i = 5 \\ (0.7644Deg_i^2 + 0.2009Deg_i + 0.0161) \cdot d_c, \; else \end{cases} \quad (17)$$

$$T = \frac{Q_{max_i} - Q_i}{\sigma \cdot \sqrt{tr_i \cdot \frac{N}{DC_i}}} \quad (18)$$

$$N_{w_i} = 1 - \sigma \cdot \sqrt{tr_i \cdot \frac{N}{DC_i}} \cdot \frac{\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(T)^2}{2}}}{1 - \frac{1}{2}(1 + 0.196854 \cdot T + 0.115194 \cdot T^2 + 0.000344 \cdot T^3 + 0.0019527 \cdot T^4)^{-4}} \quad (19)$$

The objective function is solved while respecting the following constraints on the inventory control policy. Equations 20-21 impose that the reorder point (RP_i) associated with the perishable product in each DC guarantees the expected service level. Meanwhile, Equation 22 reduces spoiled products by imposing that, for each replenishment order, the reorder quantity will be received by suppliers, stored in DCs, and distributed to customers in a period shorter than the shelf life (as outlined in Figure 2). Deepening Equation 22, it is computed following Equations 23-27. Specifically, the optimal reorder quantity is first determined by applying Wilson’s law (Q_i, Equation 23). Then, the obtained value is adjusted (Q_i^*) to remain below a threshold (Q_{max_i}, Equation 24), which depends on the product's shelf life. If Q_{max_i} in Equation 24 is negative, according to Figure 2 this means that the considered SC configuration (i) is unacceptable since any purchased product would expire before being distributed to customers. To prevent that SC configuration from being selected by the objective function, the value of Q_i^* in Equation 22 is forced equal to infinity, making the related total logistic distribution cost inconvenient (Equation 3). Without this adjustment, the negative value of Q_{max_i} would result in negative cost items, thus reducing the total logistic distribution cost (C_{TOT_i}). Therefore, an unacceptable SC configuration could appear as optimal. If all SC configurations in Figure 1 are characterized by negative Q_{max_i}, this means that the chosen input parameters (Table I) are not adequate, but this scenario is rare among all possible input parameter combinations.

$$SS_i = Z \cdot \sigma \cdot \sqrt{L \cdot \frac{N}{DC_i}} \quad (20)$$

$$RP_i = D_i \cdot L + SS_i \quad (21)$$

$$Q_i^* = \begin{cases} Q_i, if \; (L + ts_i + tt_i) \leq m \\ Q_{max_i}, if \;\; (L + ts_i + tt_i) > m \; and \; Q_{max_i} > 0 \\ +\infty, \; else \end{cases} \quad (22)$$

$$Q_i = \sqrt{\frac{2 \cdot D_i \cdot o}{h \cdot c}} \tag{23}$$

$$Q_{max_i} = \left(m - L - \frac{d_i}{v}\right) \cdot D_i - SS_i \tag{24}$$

$$ts_i = \frac{Q_i}{D_i} + \frac{SS_i}{D_i} \tag{25}$$

$$tt_i = \frac{d_i}{v} \tag{26}$$

$$tr_i = L + ts_i \tag{27}$$

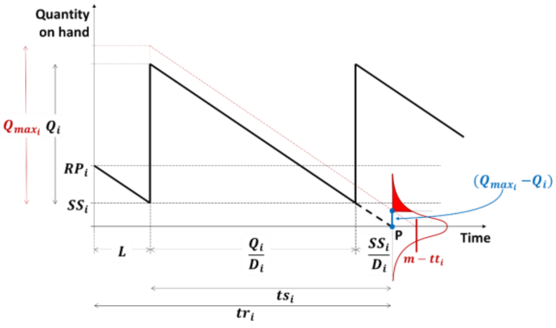

Figure 2. Inventory control policy for each product in each DC

IV. CASE STUDY APPLICATION

The proposed mathematical model underwent testing in a real-world case study to prove its practical applicability. We investigated an agri-food distribution company that had 200 DCs sited close to its main customers. Upon observing a significant decrease in customer demand for two perishable products (i.e., milk and rice), the company asked our assistance in reviewing the SC configuration for these products. Currently, the company employed a decentralized SC configuration for all products, storing stocks of milk and rice in all DCs. By using the mathematical model, we aimed to assess whether the current SC configuration was optimized or if it required revisions. First, input data were collected by consulting company databases, resulting in Table II. Next, for each perishable product, the mathematical model was applied to identify the optimal SC configuration. The obtained results are shown in Figure 3, referring to milk (up) and rice (down), respectively. Figure 3 depicts the total cost of different SC configurations, explaining how the cumulative of all cost items in Table 1 changes when Deg_i varies. According to Figure 3, decentralization ($i = 1$) appeared as the optimal SC configuration for milk, with a minimum total cost of 10,583,972 €/year. Instead, centralization ($i = 5$) emerged as the optimal SC configuration for rice, with a minimum total cost of 6,049,609 €/year. Further analysis of Figure 3 reveals the following insights. First, Figure 3 reveals the importance of introducing the constraint in Equation 22 into the mathematical model. In fact, in the case of milk, due to a reduced shelf life (m), Equation 22 highlights decentralization as the only viable SC configuration while preventing other alternatives from being selected (with a total cost of +∞). The latter SC configurations would have resulted in excessive product spoilage, leading to customer dissatisfaction and poor company performance. On the contrary, for rice, all SC configurations are feasible, confirming that the constraint in Equation 22 is more stringent for products with lower shelf life. Moreover, Figure 3 confirms that decentralization is the optimal SC configuration for products with short shelf lives, confirming the advantage of keeping inventories close to customers to reduce transportation and waste costs. In contrast, a greater degree of centralization is convenient when products have longer shelf lives, allowing to benefit from risk-pooling to contain ordering, backorder, and holding costs. Concerning how to review the SC configuration, as shown in Figure 3, the mathematical model suggests keeping unchanged the current management of milk, allocating stocks in all 200 DCs. Whereas in the case of rice, the mathematical model suggests reviewing the SC configuration opting for centralization and storing rice in a single central DC. Moreover, Figure 3 proves the importance of considering not only holding and transportation costs but also waste, backorder, ordering, and purchase costs when evaluating different SC configurations. There cost items are clearly non-negligible. Although the purchase and backorder costs are not differential when Deg_i varies, their inclusion is essential for estimating the total logistic distribution cost. Finally, Figure 3 illustrates the benefits of reviewing the SC configuration in distribution companies for perishable products, showing the economic benefits achievable and underlying the importance of the proposed mathematical model.

TABLE II
CASE STUDY INPUT DATA

Input data	Milk	Rice	Unit measure
N	200	200	-
SL	0.95	0.92	-
m	0.04	1.48	years
$\bar{D}$	45,500	20,000	units/year
σ	3,000	1,300	units/year
b	1.86	1.20	€/backorder
L	0.003	0.011	years
c	0.9	1.4	€/unit
o	5.0	5.0	€/order
h	0.30	0.20	years^{-1}
w	0.6	1.6	€/unit
d_{c_i}	25.0	25.0	km
t_i	0.7	0.7	€/km*vehicle
v	45.0	45.0	km/h

q	500	1,500	units/demand
C	10,000	10,000	units

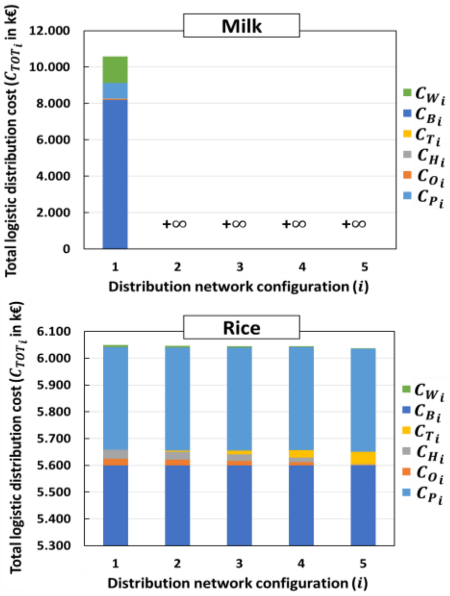

Figure 3. Total logistic distribution cost of milk (up) and rice (down) in different SC configurations

V. CONCLUSIONS

This paper presents a novel mathematical model, to review the SC configuration of distribution companies handling perishable products. The model represents the main contribution of this study, offering both theoretical and practical implications. At a theoretical level, it quantitatively compares the performance of centralized, decentralized, and hybrid SC configurations, associating individual perishable products with the most cost-effective option. At a practical level, it enables distribution companies to align logistic activities with the ever-changing customer needs, optimizing the performance by ensuring high service levels while minimizing logistic costs. Unlike existing literature, the proposed mathematical model investigates holding and transportation costs, as well as ordering, backorder, and waste costs of spoiled products. The mathematical model's effectiveness was tested on an agri-food company, reviewing the SC configuration of two perishable products. The results confirmed the applicability of the mathematical model in real-world companies and reinforced the benefits of adopting decentralized SC configurations for products with shorter shelf lives. However, it is important to acknowledge some limitations of the proposed model. It relies on simplifying assumptions, that restrict the generalizability of its results. Moreover, to facilitate strategic (long-term) decision-making on the SC configuration, the model uses annual average values (e.g., d_{c_i}, $\bar{D}$, and σ) obtained by assuming specific probability distributions. To address these limitations, as a future development of this study, we propose to remove some of the simplifying assumptions. For instance, it is possible to consider stochastic lead times, lateral transshipments, and capacity constraints in DCs. Additionally, modifying the mathematical model by considering a multi-objective function could enable the optimization of environmental and social aspects alongside the economic one, thereby improving companies' sustainability in line with the triple bottom line approach. Finally, a sensitivity analysis could be performed to improve the understanding of the results and better appreciate which are the main variables to concentrate on when selecting the optimal SC configuration.

Appendix A.

Appendix A explains how Equations 18-19 were obtained. Based on the simplifying assumptions in Section II, the customer demand (d) for the perishable product is a normal distribution ($\aleph$, Equation A1) with an expected value $\bar{D}$ and a standard deviation σ. According to Figure 2, the demand received by each DC (d_{DC}) during the period between the expected consumption of one replenishment lot and the next one (tr_i, Equation A2) is a normal distribution like in Equation A3. Consequently, the inventory (Inv_i) that remains in each DC after the period tr_i corresponds to Equation A4, being the difference between the on-hand quantity received according to the optimal reorder quantity and the demand experienced during tr_i. Correspondingly, the inventory during the period tr_i (inv_{DC,tr_i}) follows a normal distribution (Equation A5), where the expected value (Equation A6) is zero since d_{DC,tr_i} behaves as expected in Equation A3 while the standard deviation (Equation A7) is the same as in Equation A3 since inv_{DC,tr_i} is a function of d_{DC,tr_i}. In this context, Figure A1 suggests that stocks of the perishable product will spoil in a DC if the inventory at the end of period tr_i is non-null and the stocks left in inventory are replenished (L), stored (ts_i), and distributed to customers (tt_i) in an amount of time exceeding the shelf life (m). Therefore, N_{w_i} (Table I) is given by Equation A8, corresponding to the red area in Figure 2. However, according to the probability theory, N_{w_i} can be approximated as the conditional expectation of the normal variable inventory (inv_{DC,tr_i}) to be higher than the limit value which results in spoiled products (i.e., $Q_{max_i} - Q_i$). Therefore, Equation A8 can be rewritten as in Equation A9. Equation A9 is transformed into Equation A10 if the normal distribution inv_{DC,tr_i} is standardized (z). Next, Equation A10 becomes Equation A11 by expanding the terms inside the integrals, where $\varphi(z)$ is the probability density function of a standard normal distribution. Subsequently, Equation A11 is transformed in Equation A12 by considering the relationships between $\varphi(z)$ ant the cumulative distribution function $\Phi(z)$, where the apostrophe is the derivative operator. By approximating the integral $\Phi(T)$ with an arithmetic series and remembering that μ_{inv_i} is zero (Equation A6), Equation A12 becomes Equation A13, thus proving how Equations 18-19 were obtained.

$$d \sim \aleph(\overline{D}, \sigma) \qquad \text{(A1)}$$

$$tr_i = ts_i + L \qquad \text{(A2)}$$

$$d_{DC,tr_i} \sim \aleph\left(Q_i + SS_i + RP_i, \sigma \cdot \sqrt{tr_i \cdot \frac{N}{DC_i}}\right) \qquad \text{(A3)}$$

$$Inv_i = Q_i + SS_i + RP_i - d_{DC,tr_i} \qquad \text{(A4)}$$

$$inv_{DC,tr_i} \sim \aleph\left(\mu_{inv_i}, \sigma_{inv_i}\right) \qquad \text{(A5)}$$

$$\mu_{inv_i} = 0 \qquad \text{(A6)}$$

$$\sigma_{inv_i} = \sigma \cdot \sqrt{tr_i \cdot \frac{N}{DC_i}} \qquad \text{(A7)}$$

$$N_{w_i} = \int_{-\infty}^{Q_i - Q_{max_i}} \left[inv_{DC,tr_i} \cdot f\left(inv_{DC,tr_i}\right)\right] dinv_{DC,tr_i} \qquad \text{(A8)}$$

$$N_{w_i} = E\left(inv_{DC,tr_i} \middle| inv_{DC,tr_i} > Q_{max_i} - Q_i\right) \qquad \text{(A9)}$$

$$N_{w_i} = 1 - E\left(\sigma_{inv_i} \cdot z + \mu_{inv_i} \middle| z \leq T\right) \; with \; T = \frac{\left(Q_{max_i} - Q_i\right) - \mu_{inv_i}}{\sigma_{inv_i}} \qquad \text{(A10)}$$

$$N_{w_i} = 1 - \frac{\left(\sigma_{inv_i} \cdot \int_{-\infty}^{T} z \cdot \varphi(z) dz + \mu_{inv_i} \cdot \int_{-\infty}^{T} \varphi(z) dz\right)}{\Phi(T)} \; with \; T = \frac{\left(Q_{max_i} - Q_i\right) - \mu_{inv_i}}{\sigma_{inv_i}} \qquad \text{(A11)}$$

$$N_{w_i} = 1 - \frac{\left(-\sigma_{inv_i} \cdot \int_{-\infty}^{T} \varphi\prime(z) dz + \mu_{inv_i} \cdot \int_{-\infty}^{T} \Phi\prime(z) dz\right)}{\Phi(T)} \; with \; T = \frac{\left(Q_{max_i} - Q_i\right) - \mu_{inv_i}}{\sigma_{inv_i}} \qquad \text{(A12)}$$

$$N_{w_i} = 1 + \sigma_{inv_i} \cdot \frac{\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(T)^2}{2}}}{1 - \frac{1}{2}\left(1 + 0.196854 \cdot T + 0.115194 \cdot T^2 + 0.000344 \cdot T^3 + 0.0019527 \cdot T^4\right)^{-4}} \; with \; T = \frac{\left(Q_{max_i} - Q_i\right)}{\sigma_{inv_i}} \qquad \text{(A13)}$$

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