An energy saving scheduling MIP formulation for the iron-casting processes in foundries

Andrea Rossi*, Lorenzo Tiacci* and Ivan Stefani**

** Dipartimento di Ingegneria., University of Perugia, Via Duranti 97, 06125 – Perugia – Italy (andrea.rossi16@studenti.unipg.it, lorenzo.tiacci@unipg.it)*

*** Fonderie di Assisi Spa– Assisi (PG) – Italy (ivan.stefani@fonderiediassisi.com)*

Abstract: The sand-casting production process is a type of operation that provides high-energy consumption, so that even a small percentage of energy saving brings to a considerable cost reduction. This work presents a Mixed Integer Programming formulation to optimize the scheduling in cast-iron foundries with two parallel melting lines and a casting line, with the objective to minimize the energy consumption of the furnaces and the waste that occurs when the molten material prepared is not used for casting but is solidified again. The sequence of products entering the process is crucial to minimize these wastes, because the various types of alloys that constitute the different products cannot be mixed, in order not to contaminate each other. Therefore, if there is a residual liquid in the furnace, the residual must be solidified and extracted from the furnace if the subsequent alloy to be processed is different from the residual.

Keywords: energy saving scheduling, MIP formulation, iron-casting

I. INTRODUCTION

Scheduling operations is a practice that has traditionally focused on improving production efficiency, minimizing maximum completion time, production costs and delivery rate.

In recent times, the energy market evolution provided a relevant increase of energy cost due to growing geopolitical tension and limited resources availability. For this reason, in recent years, energy costs have taken an increasingly central role in management control. Therefore, scheduling models that consider the minimization of energetic costs in the objective function has been introduced.

The realization of energy savings has become a hot topic particularly in companies that have both high variety of machines and products, and processes that requires high energy consumption. The sand-casting process in foundries, consisting in melting raw materials and pouring inside vessels (called 'moulds') to obtain foundry blanks, falls in this category.

The mix of products that a foundry can produce is typically wide, thanks on one hand to the possibility of producing several types of alloys with different mechanical characteristics, and on the other hand to the ability to obtain shapes of varying weight and geometry.

The melting process of raw materials is the phase of the entire process requiring the higher amount of energy. The amount and cost of the energy needed to perform the process depend on:

- the type of furnace used for melting;
- the ability to utilize the melted liquid by limiting the so called 'residual'.

In many foundries the melting process is performed by two types of furnaces: rotary oxy-combustion furnaces and electric induction furnaces. Rotary furnaces are characterized by a relatively high speed of melting a fixed amount of raw materials at low energy cost, because the energy source is methane. On the contrary, electric furnaces have a flexible capacity and performs a slower melting at relatively higher costs.

Residual is melted liquid that must be solidified again. In fact, as we will describe in more detail in the conceptual model of the problem, depending on how the sequence of items entering the process is scheduled, it is not always guaranteed that all the available liquid from a furnace can be completely poured into moulds. This is essentially due to the fact that it is not possible to mix different types of alloys inside the furnaces. Residual represents a waste in terms of energy (because it has to be melted again to be utilized) and in terms of efficiency (because when the residual is melted again, oxides and scrapes, that reduce the amount of quantity available for pouring, are produced).

II. OBJECTIVE OF THE WORK

In the present work a Mixed Integer Programming formulation is presented for the scheduling problem in foundries with one casting line and multiple melting lines. As objective we consider the minimization of differential costs of energy related to the type of furnace utilized (rotary and electric) and the residuals created during the process.

As it will be detailed in the literary review, the contribution to the literature of the present work consists in simultaneously taking into consideration three fundamental aspects: the performance aspect, consisting in demand fulfilment in the planning horizon; the minimization of energy costs related to different types of furnaces utilization; the minimization of energy waste due to residuals.

The paper is organized as follows. In Section III the literary review is performed and the contribution to the literature of the presented work is highlighted. In Section IV the conceptual model of the problem taken into consideration is described. The MIP formulation of the model is proposed in Section V. The models are applied on an instance coming from a real case study, described in Section VI. In Section VII the results of the experiment are shown and discussed. In Section VIII further improvements of the work are proposed and conclusions are drawn.

III. LITERATURE REVIEW

One of the first paper dealing with foundry operations scheduling is the one by (Van Voorhis et al., 2001) in which an integer programming model is described that minimizes a comprehensive cost function that includes the costs of pattern tooling set-up, late delivery, WIP inventory, and under-utilization of assets. Although preceding works appeared concerning with melting and pouring, they did not consider the issue of scheduling the operations of a foundry that is producing a wide variety of alloys in a wide range products.

Since this paper, other works related to scheduling in this type of foundries appeared in the literature (Yang and Park, 2009) (Deb et al., 2003)(Stawowy and Duda, 2020, 2013)(de Araujo et al., 2008)(Bewoor et al., 2018)(Tang et al., 2022). However, none of these papers consider the minimization of costs due to energy expenditure.

The first paper that take into consideration this aspect is the one by (Haït and Artigues, 2009). They consider the integration of energy-related constraints into scheduling for a foundry, even accounting for human resource constraints. In particular, in their model two casting lines served by human operators are considered, and energy wastes occur when melting is complete, but the temperature must be hold in the furnace until an operator is ready to unload it.

(Esteban and Penya, 2012) presented a paper dealing with scheduling in foundries in which the differential costs of energy come from the period during the day in which the melting is performed. They observed that, in the foundry industries, there exists a general knowledge about which are the least expensive hours in which it would be appropriate to execute the melting process, but there is no clear control whatsoever whether that is the best solution to approach. Their approach does not consider a variety of products made by different alloys, and the connected residuals that can occur.

Other works in the literature (Lu and Qiao, 2022)(Pan et al., 2022) considering energy saving scheduling in foundries are related to the continuous casting process. The main difference with respect to the sand casting is that the molten material is not poured into single moulds, but is solidified on a continuous line into a "semifinished" billet, bloom, or slab. In this type of process there is not any alloy changeover that may cause residual.

The contribution of the present work is the formulation of a MIP model to solve a scheduling problem for sand casting foundries that produce a high variety of products made from different alloys with the objective of minimizing costs associated to residuals and differential costs of melting in different types of furnaces.

IV. THE CONCEPTUAL MODEL

The proposed model has been inspired from a real case study, related to Fonderie di Assisi spa, a Foundry located in the Umbria region of Italy producing iron casted components for the automotive industry. The conceptual model of the foundry is depicted in Fig. 1, and provides two rotary oxy-combustion furnaces (R1 and R2), two electric induction furnaces (E1 and E2), two pouring furnaces (P1 and P2), and a casting line.

The process can be described as follows. Raw materials are charged into rotary furnaces, that are able to quickly melt (approximately 2,5 h) the materials, creating a certain type of alloy. Rotary furnaces must always be filled to their maximum effective capacity of 16 tons, in order to maximize the melting energy efficiency.

The melted liquid coming from a rotary furnace is spilled into a dedicated electric furnace, in which the composition of the alloy is corrected and made ready to be poured. This phase lasts 1h. After this phase the liquid contained in the electric furnace is continuously spilled through ladles to a dedicated pouring furnace, from which the moulds in the casting line are filled. The rate of this operation depends on the type of item produced. In fact, the time to prepare and fill the mould in the casting line is different for each kind of item, determining the rate of the casting line.

Thus, there are two separated and parallel ‘melting lines’ each one composed by one rotary furnace, one electric furnace and one pouring furnace. The two pouring furnaces operate alternatively to realize the casting: while one pouring furnace is pouring the liquid prepared in the electric furnace into the moulds, the other one is idle. When the liquid contained in the electric furnace of one melting line has been completely spilled into its pouring furnace, the other melting line must be ready to pour liquid in the casting line, in order to achieve a continuous production. For this reason, because the total time needed to prepare the alloy in the rotary and in the electric furnaces of each melting line is 3,5 h, the time elapsed between two consecutive pouring from the same melting line cannot be lower than 3,5 h. Furthermore, because the time required in each electric furnace equals 1h, the duration of each pouring must always be greater than 1h, otherwise there is no time to prepare the alloy.

Each mould requires a certain amount of liquid to be spilled. Thus, the liquid contained in an electric furnace may not be completely poured, because the ‘residual’ liquid is not sufficient to fill an entire mould. The residual may follow two alternative ways: it can be utilized if the next type of alloy prepared from its melting line is the same; otherwise, it must be solidified, representing a waste in terms of energy (because it has to be melted again to be utilized) and in terms of efficiency (because when the residual is melted again, oxides and scrapes, that reduce the amount of quantity available for pouring, are produced).

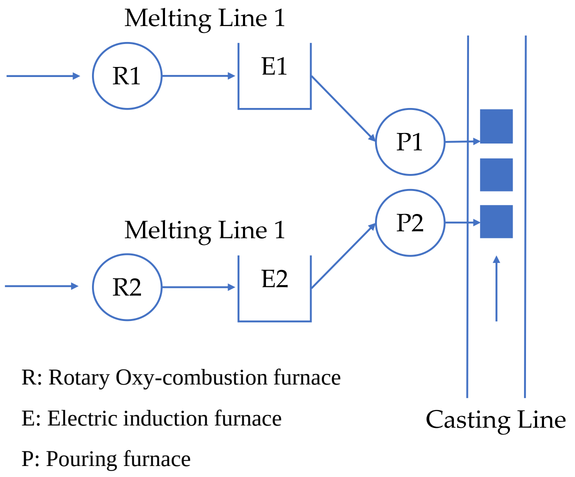

Fig. 1 The conceptual model of the foundry

The scheduling horizon is one week (5 working days). The week is divided into days. There is one shift of 9.5 h/day. However, during the night, the two electric furnaces are actives. So, in each melting line, each electric furnace is filled with melted liquid from the corresponding rotary furnace. Furthermore, there is the possibility to melt additional raw materials directly into the electric furnaces. In fact, the capacity of the electric furnaces is up to 24 tons. Melting raw materials in electric furnaces is much slower and more costly than in rotary furnaces but gives the possibility to increase the available liquid for the first pouring of the day. In summary, the initial quantity of liquid available for the first pouring of each line is flexible and can be set to reduce possible residuals.

The sequence of items produced by the casting line determines the sequence and the quantity of the different alloys prepared by the two melting lines, that in turn determines the residuals in the electric furnaces, and the quantity of raw materials melted during the night. Melting in the electric furnaces during the night, which is more expensive, and the solidified residuals after each pouring, which as above mentioned represents an energetic waste, are responsible of differential costs.

The objective of the problem is to find the schedule of items produced by the casting line that minimize these differential costs.

V. THE MIP MODEL FORMULATION

In this section two different MIP model formulations to solve the problem are presented.

The first formulation, named ‘complete model’, refers to the conceptual model described in Section IV. This model contains nonlinear objective function and constraints.

The second formulation, named ‘linear model’, provides the linearization of the objective function and the elimination of nonlinear constraints. As it will be described, this simplification results in considering that all the residuals must be solidified.

A. *The complete model*

Indexes

i = 1,…,I index for items

d = 1,…,D index for days

h = 1,…,H index for daily pouring

k = 1,…,K index for alloys.

Parameters

$TMelt$ = total time to prepare the alloy in a melting line (rotary + electric furnace) = 3.5h

$TElec$ = time to correct the alloy in the electric furnace = 1

$TDAY$ = daily work shift duration = 9.5 h

Q_i = weekly demand for item i [n. of moulds]

W_i = weight for item i [kg/mould]

TC_i = pouring cycle time for item i [h/mould]

$AL_{i,k}$ = 0; 1 if item i is made of alloy k

R = capacity of rotary furnaces = 16 tons

E = capacity of electric furnaces = 24 tons

CDN = differential cost for melting raw materials in electric furnaces during the night = 27.9 €/tons

C = cost for residuals solidification = 150 €/tons.

Decision Variables

$x_{i,d,h}$ = items i produced in pouring h of day d [moulds]

$y_{d,h,k}$ = 0; 1 if alloy k is produced for pouring h in day d

$z_{d,h,k}$ = 0; 1 if the alloy of pouring h is different from the alloy of the successive pouring (h+2) of the same melting line

$l_{d,h,k}$ = liquid of alloy k available in the electric furnace for pouring h of day d [tons]

$n_{d,h,k}$ = solidified residual of alloy k after pouring h of day d [tons]

$SP_{d,h}$ = starting time of pouring h of day d [h]

$TP_{d,h}$ = total duration of pouring h in day d [h].

Objective function

$$\min \sum_{d=1}^{D}\sum_{k=1}^{K}\sum_{h=1}^{H}\left[z_{d,h,k} n_{d,h,k} C + \left(l_{d,h,k} - y_{d,h,k} R \right) CDN \right] \qquad (1)$$

Constraints

$$\sum_{d=1}^{D}\sum_{h=1}^{H} x_{i,d,h} = Q_i \qquad \forall i \tag{2}$$

$$TP_{d,h} = \sum_{i=1}^{I} x_{i,d,h}\cdot TC_i \qquad \forall d,h \tag{3}$$

$$SP_{d,1} = 0 \qquad \forall d \tag{4}$$

$$SP_{d,h} = SP_{d,h-1} + TP_{d,h-1} \qquad \forall d,\ h = 2,\dots,H \tag{5}$$

$$SP_{d,h} - SP_{d,h-2} \ge TMelt\cdot\sum_{k=1}^{K} y_{d,h,k} \quad \forall d,k,\ h = 3,\dots,H \tag{6}$$

$$Tp_{d,h} \ge TElec\cdot\sum_{k=1}^{K} y_{d,h,k} \qquad \forall d,\ h = 2,\dots,H \tag{7}$$

$$SP_{d,h} + TP_{d,h} \le TDAY \qquad \forall d,h \tag{8}$$

$$\sum_{i=1}^{I}\frac{x_{i,d,h}\cdot W_i\cdot AL_{i,k}}{1000} \le l_{d,h,k} \qquad \forall d,h,k \tag{9}$$

$$l_{d,h,k} \ge y_{d,h,k}\cdot R \qquad \forall d,k,h = 1,2 \tag{10}$$

$$l_{d,h,k} = R\cdot y_{d,h,k} + z_{d,h-2,k}\cdot n_{d,h-2,k} \quad \forall d,k,\ h = 3,\dots,H \tag{10.1}$$

$$l_{d,h,k} \le y_{d,h,k}\cdot E; \qquad \forall d,\ h,\ k \tag{11}$$

$$n_{d,h,k} = l_{d,h,k} - \sum_{i=1}^{I}\frac{S_{i,d,h}\cdot P_i\cdot AL_{i,k}}{1000} \qquad \forall d,\ h,\ k \tag{12}$$

$$\sum_{k=1}^{K} y_{d,h,k} \le 1;\ \forall d,\ h \tag{13}$$

$$z_{d,h-2,k} \le y_{d,h-2,k} - y_{d,h,k} \qquad \forall d,k,\ h = 3,\dots,H \tag{14}$$

$$z_{d,h,k} = 1 \quad \forall d,k,\ h = H-1,H \tag{15}$$

The objective function (1) aims to minimize the differential cost introduced in Section IV. The first row in the equation expresses the cost associated to solidified residuals. The second row contains the costs associated to melting materials during the night in the electric furnaces. Constraint (2) guarantees the satisfaction of the weekly demand for each item. Constraints from (3) to (8) represent time constraints: (3) determines the duration of each pouring; (4) sets the starting time of each shift to 0 and (5) guarantees that the casting line realizes a continuous production, that is, the starting time of each pouring equals the ending time of the previous one. Constraint (6) assures that each melting line has enough time to completely prepare the molten alloy before the successive pouring from the same melting line. Constraint (7) is similar to (6) but is referred to each electric furnace, that need at least 1 h to prepare the alloy, so the duration of the pouring realized by the other melting line has to be greater than 1 h. Constraint (8) force the total duration of all the pouring realized in a shift to be less than the shift duration.

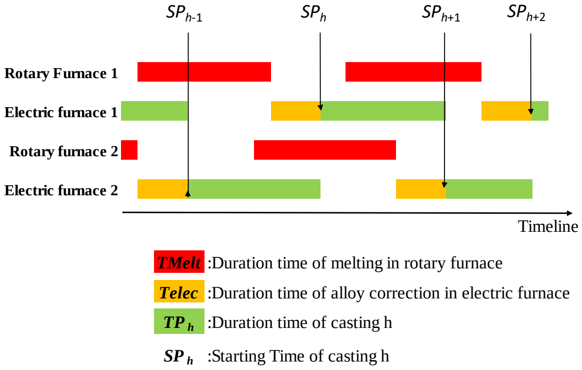

Fig. 2 Melting and pouring timeline

Constraint (9) limits the quantity of liquid utilized in each pouring to the quantity of liquid available in the electric furnace. This quantity is determined through constraints (10) to (11). It will be at least equal to the capacity of rotary furnace (10). In case there is residual of the same alloy from the previous pouring of the same melting line (note the index *h*-2 due to the alternative usage of melting lines), the residual is added to the available liquid from the rotary furnace (10.1). For the first two castings the maximum capacity of electric furnace capacity is higher, because additional raw materials can be melted in the electric furnaces during the night (11). Constraint (12) expresses the residuals after each pouring. Constraint (13) force each pouring to utilize the same type of alloy. Constraint (15) determines if there is an alloy changeover between two consecutives pouring of each melting line. Thus, even in this case, to express the alternation of the line that realize the pouring, the comparison must be made between pouring *h* and the previous pouring by the same melting line, *h*-2.

B. *Linearization of the model*

In the ‘complete model’ described in Section A, the objective function (1) and the constraint (10.1) are nonlinear. In order to provide a model that is easier to solve, in this section we propose a linear model, that, with respect to the complete model, presents the following difference: all the residuals are solidified, that is, they cannot be re-utilized even if the next pouring from the same melting line has the same type of alloy.

In this way, the decision variable $z_{d,h,k}$ is no longer needed, so as constraints (14) and (15), and the objective function (1) becomes:

$$\min\sum_{d=1}^{D}\sum_{k=1}^{K}\left(\sum_{h=1}^{H} n_{d,h,k}\cdot C + \left(l_{d,h,k} - y_{d,h,k}\cdot R\right)\cdot CDN\right) \tag{1.1}$$

Furthermore, constraint (10.1) becomes:

$$l_{d,h,k} = y_{d,h,k}\cdot R \qquad \forall d,k,\quad h = 3,\dots,H \tag{10.2}$$

VI. Design of Experiment

The two models have been implemented through the software Xpress-IVE from FICO^{TM} Xpress Optimization

Suite, which implements solving algorithms primarily adopting the Branch and Bound method. The instance analysed (see Fig. 3) provides 26 items (I = 26) made from 4 types of alloys (K = 4). For each item, input data relates to the number of moulds to be produced in the planning horizon (Q_i), the weight of the liquid needed to fill each mould (W_i), and the time needed to fill a single mould of that item in the casting line (TC_i) (whose inverse is the production rate of the casting line for that item). The maximum number of pouring per day is five (H = 5), while the scheduling horizon is five working days (D = 5). The other input values of the parameters have been specified in Section V.

Even setting a high computational time (more than 1h), the solver has not been capable to find a feasible solution for the complete model. Thus, results are reported just for the linear model.

The results found by the solver has been compared to the scheduling adopted by the company for the same instance.

Item	Q_i [n° molds]	W_i [kg/mold]	Tc_i [h/mold]	Alloy
1	750	80.5	0.0074	1
2	70	81.92	0.0076	3
3	70	18.37	0.0074	3
4	50	72	0.0074	1
5	150	71	0.0074	3
6	200	83.2	0.0074	2
7	250	103	0.0074	2
8	200	65	0.0074	2
9	100	57.2	0.0074	2
10	200	71	0.0098	2
11	400	90.8	0.0091	2
12	90	55.3	0.0108	2
13	200	99.75	0.0086	2
14	150	89.1	0.0074	1
15	500	64.44	0.0078	3
16	500	45.3	0.008	2
17	100	75	0.008	1
18	80	118.4	0.008	2
19	320	100.96	0.008	2
20	50	108	0.0297	1
21	250	116.2	0.0088	2
22	510	137	0.0108	1
23	18	35	0.0077	1
24	45	28	0.0077	1
25	20	20	0.0077	1
26	40	160	0.0088	1

Fig. 3 The instance

VII. RESULTS

The linear model can be solved in an acceptable amount of time (10 mins). Although the solution is not optimal, it is very near to the lower bound found by the solver (gap = 1.5 %). In Fig. 4 it can be seen as the solver succeeds to reduce very quickly the gap from the gap from the lower bound, and then requires much more time improve the solution.

Analysing the solution (see Fig. 5) it is evident how the solidified residuals are almost eliminated. Even melting during the night is very limited. Recall that melting overnight, although is more costly, allows to fill the electric furnace with the exact quantity needed to not produce residuals.

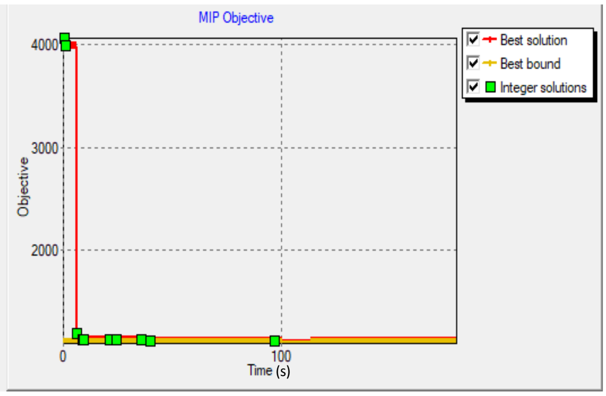

Fig. 4 The evolution of the objective function value

Table I shows the comparison between the differential costs of the scheduling obtained through the solver and the scheduling adopted for the same instance by the company. From results is evident that the value of the objective function found by the solver is approximately 55% lower than the one related to the scheduling of company. While the gap between the differential costs during the night is not too marked, the gap between the costs of solidified residues, which in the scheduling of the solver were close to 0, is much more relevant. The total savings per week equals about 1600 €/week, which is considered relevant for the company.

TABLE I
RESULT COMPARISON

Tipo	**Solidified Residual [tons]**	**Costs of residuals**	**Melting night costs**	**OBJ**
Solver	0.14	34.239 €	1'343.66 €	1'377.9 €
Company	9.7	1'455.00 €	1'553.00 €	3'008.00 €

VIII. CONCLUSIONS AND FUTURE IMPROVEMENTS

The results obtained applying the proposed model to the real case study are satisfying, even if they come from the linearized model.

The limits of the proposed approach consist in:

- Because the complete model is not solvable, further cost savings are precluded, due to the impossibility to consider residual reutilization.
- The number of shifts in which the demand must be satisfied, that is connected to the total amount of time available to satisfy the demand, is required as input of the model.

Further improvements of this work will consist in developing a meta-heuristic procedure that considers all the features of the complete conceptual model, and that allows finding near-optimal solutions by also minimizing the total completion time of operations.

IX. ACKNOWLEDGMENTS

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Appendix A. FIRST APPENDIX

n° Day	n° Casting in the day	Item id	Alloy	n° Moulds	Start Time	End Time	Poured Weight	Cumulative Weight	Melted overnight	Solidified Residual
1	1	22	1	175	0.00	1.89	23.98	23.98	7.98	0
1	2	13	2	188	1.89	3.50	18.00	18.00	0.00	0
1	2	21	2	42	3.50	3.87	4.88	22.88	0.00	0
1	2	6	2	1	3.87	3.88	0.08	22.96	6.96	0
1	3	6	2	10	3.88	3.95	0.83	0.83	0.00	0
1	3	8	2	1	3.95	3.96	0.07	0.90	0.00	0
1	3	16	2	333	3.96	6.62	15.08	15.98	0.00	0.0181
1	4	14	1	94	6.62	7.32	8.38	8.38	0.00	0
1	4	17	1	1	7.32	7.33	0.08	8.45	0.00	0
1	4	24	1	40	7.33	7.64	1.12	9.57	0.00	0
1	4	25	1	1	7.64	7.64	0.02	9.59	0.00	0
1	4	26	1	40	7.64	7.99	6.40	15.99	0.00	0.0096
1	5	6	2	25	7.99	8.18	2.08	2.08	0.00	0
1	5	8	2	7	8.18	8.23	0.46	2.54	0.00	0
1	5	12	2	1	8.23	8.24	0.06	2.59	0.00	0
1	5	18	2	42	8.24	8.58	4.97	7.56	0.00	0
1	5	19	2	83	8.58	9.24	8.38	15.94	0.00	0.05722
2	1	7	2	233	0.00	1.72	24.00	24.00	8.00	0
2	2	1	1	298	1.72	3.93	23.99	23.99	7.99	0
2	3	1	1	136	3.93	4.94	10.95	10.95	0.00	0
2	3	20	1	42	4.94	6.16	4.54	15.48	0.00	0
2	3	23	1	14	6.16	6.26	0.49	15.97	0.00	0
2	3	25	1	1	6.26	6.27	0.02	15.99	0.00	0.006
2	4	10	2	46	6.27	6.72	3.27	3.27	0.00	0
2	4	18	2	31	6.72	6.97	3.67	6.94	0.00	0
2	4	21	2	78	6.97	7.66	9.06	16.00	0.00	0
2	5	17	1	3	7.66	7.68	0.23	0.23	0.00	0
2	5	22	1	115	7.68	8.93	15.76	15.98	0.00	0
2	5	25	1	1	8.93	8.93	0.02	16.00	0.00	0
3	1	15	4	249	0.00	1.94	16.05	16.05	0.05	0
3	2	10	2	154	1.94	3.45	10.93	10.93	0.00	0
3	2	11	2	126	3.45	4.60	11.44	22.37	0.00	0
3	2	19	2	1	4.60	4.60	0.10	22.48	6.48	0
3	3	6	2	36	4.60	4.87	3.00	3.00	0.00	0
3	3	7	2	16	4.87	4.99	1.65	4.64	0.00	0
3	3	11	2	122	4.99	6.10	11.08	15.72	0.00	0
3	3	16	2	6	6.10	6.15	0.27	15.99	0.00	0.0074
3	4	6	2	128	6.15	7.09	10.65	10.65	0.00	0
3	4	9	2	3	7.09	7.12	0.17	10.82	0.00	0
3	4	12	2	88	7.12	8.07	4.87	15.69	0.00	0
3	4	16	2	4	8.07	8.10	0.18	15.87	0.00	0
3	4	18	2	1	8.10	8.11	0.12	15.99	0.00	0.0128
3	5	13	2	12	8.11	8.21	1.15	1.15	0.00	0
3	5	18	2	3	8.21	8.23	0.36	1.50	0.00	0
3	5	19	2	143	8.23	9.38	14.44	15.94	0.00	0.05852
4	1	15	4	251	0.00	1.96	16.17	16.17	0.17	0
4	2	1	1	129	1.96	2.91	10.38	10.38	0.00	0
4	2	4	1	50	2.91	3.28	3.60	13.98	0.00	0
4	2	14	1	56	3.28	3.70	4.99	18.97	0.00	0
4	2	24	1	1	3.70	3.70	0.03	19.00	3.00	0
4	3	7	2	1	3.70	3.71	0.10	0.10	0.00	0
4	3	9	2	96	3.71	4.42	5.49	5.59	0.00	0
4	3	11	2	108	4.42	5.40	9.81	15.40	0.00	0
4	3	16	2	5	5.40	5.44	0.23	15.63	0.00	0
4	3	18	2	3	5.44	5.47	0.36	15.98	0.00	0.0177
4	4	9	2	1	5.47	5.47	0.06	0.06	0.00	0
4	4	16	2	151	5.47	6.68	6.84	6.90	0.00	0
4	4	19	2	89	6.68	7.39	8.99	15.88	0.00	0
4	4	21	2	1	7.39	7.40	0.12	16.00	0.00	0.00086
4	5	1	1	5	7.40	7.44	0.40	0.40	0.00	0
4	5	22	1	113	7.44	8.66	15.48	15.88	0.00	0
4	5	24	1	4	8.66	8.69	0.11	16.00	0.00	0.0045
5	1	2	3	70	0.00	0.54	5.73	5.73	0.00	0
5	1	3	3	70	0.54	1.06	1.29	7.02	0.00	0
5	1	5	3	150	1.06	2.17	10.65	17.67	1.67	0
5	2	17	1	96	2.17	2.93	7.20	7.20	0.00	0
5	2	22	1	107	2.93	4.09	14.66	21.86	5.86	0
5	3	8	2	183	4.09	5.44	11.90	11.90	0.00	0
5	3	11	2	44	5.44	5.84	4.00	15.89	0.00	0
5	3	12	2	1	5.84	5.85	0.06	15.95	0.00	0
5	3	16	2	1	5.85	5.86	0.05	15.99	0.00	0.0092
5	4	1	1	182	5.86	7.21	14.65	14.65	0.00	0
5	4	20	1	8	7.21	7.44	0.86	15.52	0.00	0
5	4	23	1	4	7.44	7.47	0.14	15.66	0.00	0
5	4	25	1	17	7.47	7.60	0.34	16.00	0.00	0.005
5	5	8	2	9	7.60	7.67	0.59	0.59	0.00	0
5	5	19	2	4	7.67	7.70	0.40	0.99	0.00	0
5	5	21	2	129	7.70	8.84	14.99	15.98	0.00	0.0214

Fig. 5 The solution found by the solver