An analysis of the vehicle routing problem for logistics distribution


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Abstract: The vehicle routing problem (VRP) is a combinatorial optimization and integer-programming problem formulated to answer the following question: “What is the optimal set of routes for a fleet of vehicles to traverse in order to deliver to a given set of customers?”. In logistics distribution, VRP is frequently encountered and because it is a non-polynomial (NP)-hard problem of operational research, trying to solve it for optimality is non-trivial and there are limited instances of the problems that can actually be solved optimally. In this paper, we carried out a simulation study whose aim is to estimate the length of the optimal tour (i.e. shortest tour) bypassing the need for solving the Traveling Salesman Problem (TSP). To this end, we introduced two dimensionless coefficients (k1 and k2). Both coefficients are computed starting from a simple tour (i.e. the “round trip” tour). The coefficients describe the ratio between the length of the “round trip” tour and the tour obtained by applying the CW savings algorithm (k1) or the “optimal” tour (k2). By analyzing a number of retail stores (RS) from 1 to 11, we plan to derive a trend for k1 and k2 as a function of N so that, for higher N values, one can directly compute the length of the tour using these coefficients (k2 in particular) and the length of the round trip tour, without solving the TSP. The analysis carried out will be exploited to support the design of a reverse logistics channel to recover food waste from RSs, which is the aim of a research project currently in process at the University of Parma and targeting the Emilia-Romagna region.

Keywords: vehicle routing problem; Clarke-Wright saving algorithm; permutations; logistics distribution

1 Introduction

The Vehicle Routing Problem (VRP) plays a central role in the fields of physical distribution and logistics activities, as it involves the design of a minimum cost delivery route, starting and terminating at a depot, which services a set of customers (Je ´ bek et al., 2016). The VRP is classified as a non-polynomial (NP) hard problem (Kumar & Panneerselvam, 2012). There are several versions of the problem and a wide variety of exact and approximate algorithms have been proposed for its solution. Exact algorithms can only solve relatively small problems, but a number of approximate algorithms have proved very satisfactory (Laporte, 1992). In order to solve VRP problems from the real world more effectively, many algorithms, particularly heuristics, were designed and implemented. The saving approach proposed by Clarke and Wright (CW) is a well-known heuristic that is able to solve VRP problems relatively efficiently and is widely used to this end (Cao, 2012). Our purpose with this paper is to estimate the length of the optimal tour that can be obtained by solving the TSP problem using as input only the length of a very simple tour (i.e. the round trip tour) and the number of elements (N) to be visited during the trip.

The round trip tour (Figure 1a) is a simplified route, obtained by visiting each point starting and ending the tour at the depot. The corresponding length is, therefore, quite easy to compute. Conversely, the typical route generated when solving the VRP is expected to be more similar to the tour depicted in Figure 1b. Such a tour can be obtained either by solving the TSP for optimality (which can be typically done for small instances of the problem) or by applying a heuristic algorithm to find an effective solution of the TSP. To be more precise, in this study the length of the optimal tour has been found by analyzing extensively the whole solutions space of the TSP for small instances of the problem (i.e., N ≤ 8, which leads to 8! =40320 solutions). Such an analysis was supported by an exhaustive procedure, developed in visual basic for applications (VBA) under Microsoft Excel™, which examines all possible routes and computes their total length, thus identifying the optimal one. For higher values of N, an effective solution of the TSP has been found by applying the CW saving algorithm.

It is reasonable to expect that a relationship exists between the length of the tour obtained by solving the TSP (either as the optimal solution or as an effective approximation) and the length of the round trip tour. Indeed, both the length of the round trip tour and of the tour obtained when solving the TSP basically depend on the number of points (N) to be visited and on their location. Therefore, these lengths are compared, to derive two coefficients, called k1 and k2, which only depend on N and allows approximating the TSP solution starting from the length of the round trip tour. Overall,
the outcomes of the study should be helpful in practical cases to estimate the length of the TSP tour as a function of $N$ and of the length of round trip tour.

![Figure 1: scenarios considered in the analysis – round trip tour (a) and CW tour (b).](image)

The paper proceeds as follows. The next section presents a brief review of the literature about the problem; section 3 provides some information about the case study under examination. Section 4 describes the model developed to evaluate the optimal set of routes. Section 5 discusses the main results and Section 6 concludes by highlighting the main limitations and suggesting future research directions.

2 Literature

Dantzig & Ramser (1959) were the first authors who introduced the “Truck Dispatching Problem”, by modelling how a fleet of homogeneous trucks could fulfil the demand for oil of a number of gas stations from a central hub and with a minimum travelled distance. This problem can be considered as a generalization of the “Traveling-Salesman Problem”. Five years later, Clarke & Wright (1964) generalized this problem to a linear optimization problem that is commonly encountered in the domain of logistics and transport, i.e. how to serve a set of customers, geographically dispersed around the central depot, using a fleet of trucks with varying capacities. This was subsequently named the VRP and is currently one of the most widely studied topics in the field of operations research (Brackers et al., 2016).

The literature about the VRP includes exact methods, heuristics and meta-heuristics approaches as well as hybrid methods, i.e. a combination of exact, heuristic or meta-heuristics procedures (Kumar & Panneerselvam, 2012). The exact algorithms are only efficient for small problem instances, while heuristics and meta-heuristics are often more suitable for practical applications, because real-life problems are considerably larger in scale (Brackers et al., 2016). Most heuristic methods can be classified into two categories as follows: 1) constructive methods – taking into account capacities and costs, routes are made by adding nodes to partial routes or combining sub-routes, as it is the case for the CW algorithm; 2) two-phase methods – they consist of: a) clustering of vertices and b) route construction (Hashi et al., 2015).

The CW algorithm, originally proposed by Clarke & Wright (1964) to solve the capacitated VRPs in which the number of vehicles is free, is nowadays one of the most widely applied heuristics for solving due capacitated VRP to its simplicity of implementation and efficient calculation speed. The CW algorithm is also able to generate solutions that are nearly optimum (Jeábek et al., 2016). It has also been widely applied as a basis for many commercial packages routing algorithm (Pichpibul & Kawtummachai, 2012).

There are many papers about the application of VRP and CW algorithm in different areas: optimum routing of a fleet of gasoline de-livery trucks between a bulk terminal and a large number of service stations supplied by the terminal (Dantzig & Ramser, 1959); the VRP and scheduling issues of transportation service for Dhaka City, Bangladesh (Hashi et al., 2015); an urban-decision support system devoted to manage, in a unified framework, the logistic services of the smart cities, such as postal delivery and waste collection services (Abbatecola et al., 2016); development of a new problem framework that describes a formal method for quantitatively assessing the impact of including unverified information in disaster relief planning (Kirac et al., 2015); the introduction enhanced steps in route building, stop assignments, and route balancing is able to obtain superior results in solving VRP (Cao, 2012); Pichpibul & Kawtummachai (2012) have proposed an algorithm that has been improved from the classical Clarke and Wright savings algorithm (CW) to solve the capacitated vehicle routing problem. The main concept of their proposed algorithm is to hybrizde the CW with tournament and roulette wheel selections to determine a new and efficient algorithm. The study proposed by Segerstedt (2014) presents a variant of the Clarke and Wright's saving method that is suitable for introducing the vehicle routing problem and the importance of efficient vehicle routing. The method uses only the first pair of calculated savings and uses these also when searching for complements or additions to an already decided route.

However, although the food waste management is widely discussed in the literature (Ridoutt et al., 2010; Righi et al., 2013; Balaji & Arshinder, 2016; Sala et al., 2017; Tostivint et al., 2017; Sgarbossa & Russo, 2017), only few
studies apply the VRP to the food waste recovery issue. The study developed by Kim et al. (2016) performed computational comparisons between VRP and pollution routing problem (PRP) using case studies derived from VRP libraries and a real situation, namely the reverse logistics of disposed food waste in Seoul. The authors aim to analyse the extent to which the introduction of PRP can reduce greenhouse gas (CO₂) emissions. On the contrary, the goal of the study developed by Araujo et al. (2010) is to propose a method to evaluate the costs of biodiesel production from waste frying oils to develop an economic assessment of this alternative. To determine the logistics cost, a mathematical programming model is proposed to solve the VRP, which was applied to an important urban centre for biodiesel production and consumption in Rio de Janeiro (Brazil). In addition, the CW algorithm has not yet been applied to these problems, to our knowledge. This paper tries to contribute to the literature by addressing these gaps, in that it carries out a series of analyses to estimate an efficient path to recover food waste.

3 Context: the SORT project

The shocking report of the United Nations’ Food and Agriculture Organization (FAO) states that, every year, one third of the food produced for human consumption is lost or wasted globally (Gustavsson et al., 2011). The total food losses in Italy, every year, sum up from 10 to 20 billion tons, valued at about 37 billion euro (Carnazzi, 2015). Thus, the world has become more aware of the issue of food waste. From this evidence, a research project was started at the University of Parma in 2014. The project is called “SORT” (Italian acronym for “tecnologies and models to unpack, manage inventory and track wasted food”) and includes six partners, i.e. two universities (University of Parma and Ferrara) and four companies (food machine manufacturers and logistics operators). The general aim of the project is developing an integrated solution to manage the recovery of food waste in the food supply chain efficiently, focusing on the amount of packaged food wasted at the RSs. The area of the Emilia-Romagna region, in the north of Italy, was selected as the starting point for this analysis. Within the SORT project, the University of Parma has dealt specifically with the development of a reverse logistics network to recover food waste.

The analysis described in this paper is in line with the activities carried out by the University of Parma within the SORT project. Indeed, the design of a recovery channel for food waste can be captured effectively by a VRP where the RSs are the customers to be served and the wasted food is the product to be retrieved by them. Compared to the shipment of finished product, retrieving food waste involves significantly lower quantities of product, which range from 15 to 120 kg/day depending on the size of the RS considered. Also, the maximum number of RS to be visited by a truck in a work shift is quite limited. To be more precise, hypothesizing that the truck needs 15 minutes to reach a RS starting from the depot and 13 minutes for unloading/loading of the product pallets and administrative procedures, the maximum $N$ is obtained as follow:

$$\frac{8\ \text{h/day} \times 60\ \text{min/h}}{15+15+13 \text{min/RS}} = 11.16\ \text{RSs/day}$$

meaning that no more than 11 RSs can be visited in a day.

Even if a truck could visit only hypermarkets in a day (which is obviously not realistic), the amount of waste retrieved ($\approx 1100-1300\ \text{kg/day}$) would never saturate its capacity. Therefore, the vehicle capacity is not an issue for the problem considered.

At the time of writing, the location of the RSs from which the wasted food should be recovered is not known, as it is still not possible to know which RSs will be involved in the experimental phase of the project. Therefore, the RS location was estimated applying a probabilistic approach with numerous replicates, to ensure statistical evidence of the results. In line with this fact, for the problem in exam the key element that affects length of the two tours is the number of RSs to be visited ($N$), on which the analysis was focused.

As previously shown in Figure 1a, the round trip tour describes a situation where a truck reaches each RS from a central depot and returns to the depot. In case of food waste recovery, the depot could be, for instance, a distribution centre or a central warehouse. It is self-evident that the round trip tour is non-optimal from the point of view of the total distance covered; however, it is very easy to compute the total length of such a tour.

Conversely, the picture in Figure 1b depicts a route that is more similar to that generated when solving the VRP associated with the recovery of food waste from a set of RSs: in this situation, the vehicle will try to visit more (or all) customers within the same trip and return to the depot once these RSs have been visited. Such a route, as mentioned, can be obtained either by solving the TSP in an optimal way or by applying a heuristic algorithm to identify an effective solution.

The relationship between the two routes in Figure 1 can be described by the mathematical formulae that follow:

$$D_{\text{optimal}} = f(D_{\text{round trip}})$$

$$D_{\text{CW}} = f(D_{\text{round trip}})$$

where $D$ describes the distance (length) of the different tours.

The CW saving algorithm does not necessarily return the optimal tour, although some authors have proved this algorithm to be effective when applied to small instances of the VRP (Laporte, 1992; Cao, 2012). However, because of its ease of adoption, it is suitable to be used also to solve large instances of the TSP. For the problem under examination, we tested the performance of the CW
algorithm by comparing the solution returned with the optimal one for $N \leq 8$. Such a check should ensure that the CW algorithm could be used effectively also to solve larger instances TSP, for which the whole solutions space cannot be examined. For $N \leq 8$, the optimal tour was obtained by means of an exhaustive procedure, developed in VBA under Microsoft Excel™, which examines all possible routes and computes their total length, thus identifying the shortest one.

By this study, we aim at approximating the optimal solution of the VRP by formulating a surrogate problem, which could be useful in practice to get a preliminary idea about the total distance to be covered to retrieve the wasted food and thus to estimate the cost of reverse logistics activities.

4 Modelling scenario

4.1 Assumptions

Regardless of the algorithm applied, the number of RSs to be visited ($N$) is assumed to be among the input data of the analysis.

Conversely, the location of the RSs from which the wasted food should be recovered is not known. Indeed, more than 1500 RSs are located in the Emilia-Romagna region and at the time of writing, it is still not known which RSs will be involved in the experimental phase of the project. Therefore, in this study we adopted a probabilistic model, developed once again in VBA under Microsoft Excel™, to locate the RSs randomly in the Emilia-Romagna region, as a function of the population density (implication of this choice will be discussed in the Conclusions). The analysis was repeated for 100,000 sets of random coordinates of the RSs, to obtain a statistically significant sample and robust outcomes. Taking into account their maximum number (i.e. $N=11$), the RSs were allocated randomly into a squared area with side equal to 20 km.

The central depot is assumed to be the centre of gravity of the RSs distribution: the rationale behind this assumption is that it is reasonable to expect that a depot will manage the recovery activities of the RSs that are close to it and can be reached easily from it.

4.2 “Round trip” tour

The computation of length of the round trip tour did not require any specific algorithm. Indeed, once the positions of the RSs have been assigned by the probabilistic model developed in Microsoft Excel™, the distance of “round trip” tour is simply computed by adding up all the distances between the central depot and the RSs.

4.3 Optimal tour

For small instances of the problem ($N \leq 8$), the VRP has been solved for optimality by analysing all possible tours, given a set of $N$ nodes to visit. This means that all tours resulting from the permutations of the $N$ nodes should be generated (i.e., $N!$ tours), their length should be computed and the shortest one should be identified. An exhaustive procedure has been developed under VBA in Microsoft Excel™ to build $N!$ routes starting from the set of $N$ nodes, compute the total length covered by each path and identify the shortest one.

Besides identifying the shortest route, the VBA macro also computes the ratio between the distance covered by a vehicle that follows the “round trip” tour and the length of the optimal tour. From the ratio, a coefficient $k_i$ is computed as follows:

$$k_i = \frac{D_{\text{round trip}}}{D_{\text{optimal}}} \quad \text{[dimensionless]}$$

Obviously, the computational time to examine $N!$ tours increases rapidly with the increase in $N$. In particular, we found that the computational time is no longer acceptable with $N>8$. Therefore, the analysis of the optimal tour was limited to $N \leq 8$.

4.4 Development of the CW algorithm in VBA

The Clarke and Wright (1964) algorithm is based on the saving generated when connecting directly two nodes $(i$ and $j)$ of a network in the same path, starting from an origin $0$, instead of carrying out a “round trip” tour, as shown in Figure 2.

![Figure 2: Concept of saving.](image)

The computational procedure involves the following steps:

1. Compute the savings $S_{ij} = C_{io} + C_{oj} - C_{ij}$ for each pair of nodes $(i,j)$;
2. Order the savings in a non-increasing fashion;
3. Consider two vehicle routes containing arcs $(i,0)$ and $(0,j)$, respectively. If $S_{ij} > 0$, tentatively merge these routes by introducing arc $(ij)$ and by deleting arcs $(i,0)$ and $(0,j)$. Implement the merge if the resulting route is feasible. Repeat this step until no further improvement is possible; otherwise, stop (Laporte, 1992).
A macro in VBA was built to automate the steps of the CW algorithm described above and implement it with different values of $N$. As per the previous situation, the VBA macro computes the ratio between the distance covered by a vehicle that follows the “round trip” tour and that of a vehicle that follows the tour returned by the CW algorithm, thus computing the $k_2$ coefficient as follows:

$$k_2 = \frac{D_{\text{round trip}}}{D_{\text{CW}}} \quad \text{[dimensionless]} \quad (5)$$

Obviously, the simulation duration increases with $N$. For instance, with $N=3$ the macro completes the computation of 100,000 random set of coordinates of the RSs in 40 minutes, while with $N=11$ the analysis required approximately 20 hours.

![Figure 3: summary of the results obtained](image)

5 Results and discussion

Figure 3 summarises the findings of the study. The figures report the following outcomes:

- the average values of $k_1$, computed on the sample of 100,000 random coordinates of the RSs, for $N=1,…8$;
- the confidence interval at 95% of $k_1$ (lower bound and upper bound);
- the linear trend (with the related equation and $R^2$ value) for the average values of $k_1$;
- the predicted values of $k_1$ for $N=9, 10, 11$;
- the average values of $k_2$, computed on the sample of 100,000 random coordinates of the RSs, for $N=1,…11$.

A first consideration from Figure 3 is that $k_1$ and $k_2$ almost coincide for $N \leq 4$, confirming that the CW algorithm is able to identify the optimal solution of the TSP for small instances of the problem. For higher $N$, it is reasonable to expect that $k_1 > k_2$, indicating that the CW has returned a TSP solution that does not coincide with the optimal one. Nonetheless, the difference between these values is relatively small, ranging from 12.92% for $N=5$ to 1.92% for $N=8$. The $k_1$ values follow an almost linear trend, which is well approximated ($R^2=0.9994$) by the following formula:

$$k_1 = 0.1995(N - 2) + 1.0416 \quad (6)$$

Using eq.6, the $k_1$ values and the related confidence interval at 95% were estimated for $N=9, 10, 11$, with results again reported in Figure 3. It can be seen from the figure that $k_2$ values obtained from the VBA macro always fall into the 95% confidence interval of the estimated $k_1$ values. These results suggest that the CW saving algorithm is effective in approximating the optimal result of the TSP for the case under examination. Most importantly, using the linear approximation of $k_1$ shown in eq.6, the length of the optimal tour can always be
obtained using, as input, the length of the round trip tour and the number of RSs to be visited.

6 Conclusion
This paper has proposed an analysis of the VRP with Microsoft Excel™, whose ultimate aim was to estimate the length of the optimal tour (i.e., shortest tour) of a truck that has to retrieve wasted food from a set of N customers and return to the depot, bypassing the need for solving the TSP associate to this problem.

Starting from the consideration that the TSP is a NP-hard problem of operational research, whose solution is not trivial for large N, we took into account a simpler situation, called “round trip” tour, which describes a situation where a truck reaches each RSs from a central depot and returns to the depot. The length of this tour is quite easy to estimate. We then introduced two dimensionless coefficients \(k_1\) and \(k_2\), that express the ratio between the length of the “round trip” tour and that of the tour resulting when solving the TSP. The solution of the TSP was obtained in two ways. For small instances of the problem \(N \leq 8\), we used an exhaustive procedure developed in VBA under Microsoft Excel™, which investigates the whole solutions space and returns the optimal one; the ratio between the length of the round trip tour and the optimal one is expressed by \(k_1\). For larger instances of the problem \(N = 9, 10, 11\), the CW saving algorithm was used to identify an effective solution of the TSP, as it has been proved to perform well to solve the TSP; the ratio between the length of the round trip tour and the optimal one is expressed by \(k_2\). Higher values of N were not considered in this study, because they are not realistic for the scenario investigated. Nonetheless, by analysing the \(k_i\) values as function of N, it was found that they can be effectively approximated by a linear trend, for which the main coefficients were derived. Hence, the optimal solution of the TSP can be obtained for higher values of N using the results of this study, using only N and the length of the round trip tour as input. Overall, from a theoretical perspective the analysis developed in this paper allows to estimate the length of the optimal tour bypassing the need for solving the TSP. From a practical point of view, the estimate obtained will be useful to derive some preliminary information about the distance to be covered to recover food waste from the RSs in the Emilia-Romagna region, which is the context were this research has been carried out. From a technical perspective, some limitations of the model should be mentioned. Specifically, the model determines the best path based on a probabilistic approach, as at the time of writing the exact RSs that will be involved in the SORT project are not known. Obviously, however, the results of the analysis could be refined if considering the real location of the RSs, which is a natural future step of this research. Moreover, the present analysis does not take into account the physical infrastructures, i.e., road connections, available to connect the nodes of the network. Rather, ideal (straight) distances are considered when connecting the nodes and therefore when calculating the tour length. This is actually not expected to jeopardize the results obtained, as a ratio between two straight distances (as we computed) is not expected to be so different from a ratio between two real distances. Nonetheless, taking into account the physical road connections will be a future adjustment to be made to the model.

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